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DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

Université Paris VII
4-7 December 1984

Series of invited talks by

Professor Herbert Überall
Deparment of Physics
Catholic University of America
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DEPARTMENT OF PHYSICS

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Professor Quentin and the organizers of this colloquium have honored me by their request that I present the first scientific talk in this Colloquium on the Scattering of Ultrasonic Waves, as well as the opening talks of each session of the colloquium. I greatly appreciate this honor which permits me to present, in the surroundings of the venerable Ecole Normale Superieure of the University of Paris, to an assembly of renowned scientists and of students aspiring to such a status also, the fruits of twenty years

work on acoustic problems which I have been able to carry out with my collaborators and students at the Department of Physics of the Catholic University of America in Washington, D.C., as well as with numerous scientists in Government laboratories of the Washington area, with whom I had the good fortune to be able to collaborate. In most of my lectures, I took pains to present not only a resume of what was done by us previously, but, since we always keep on working, to include also the latest results of our studies which have not yet been published, in order to keep the participants of the colloquium informed of our present interests. Of great importance, additionally, is the inclusion in my talks of some of the results of numerous experimental studies that were carried out here, in French laboratories, studies which were induced by our earlier theories, and which they fortunately have well verified, such as the experiments of Professor Quentin, Dr. de Billy and their team at the University of Paris VII on the penetration of elastic objects by high-frequency ultrasound; and the experiments on resonance and surface waves on elastic objects carried out by Professor Ripoche, Dr. Maze and their colleagues at the University of Le Havre, supported theoretically by Drs. Derem and Rousselot of the Laboratoire Central des Telecommunications of Velizy-Villacoublay; and recently also the experiments of Drs. Gazanhes and Sessarego at the CNRS at Marseilles, and those of other scientists. We are profoundly satisfied that our theoretical suggestions have caused, if I may say so, such an echo in France. I specially hope that out of our studies on resonances, and of the related experiments, new methods and applications will be developed in the field of Acoustics.

Avant-propos.

Monsieur le Professeur Quentin et les organisateurs de ce colloque m'ont accordé le grand honneur de présenter le premier exposé scientifique de ce Colloque sur la Diffusion des Ondes Ultrasonores, ainsi que les exposés introductifs de chaque partie du colloque. J'apprécie amplement cet honneur qui me permet de présenter, dans le cadre de la vénérable Ecole Normale Supérieure de l'Université de Paris, à une assemblée de chercheurs renommés et d'étudiants qui le seront un jour également, les fruits d'un travail de vingt ans sur les problèmes d'acoustique que j'ai pu effectuer avec mes collaborateurs et étudiants au Département de Physique de l'Université Catholique d'Amérique à Washington, ainsi qu'avec de nombreux chercheurs dans les laboratoires gouvernementaux des environs de Washington avec qui j'ai eu la chance d'avoir pu collaborer.

Dans la plupart de mes exposés, je me suis donné la peine de non seulement présenter un résumé de ce qu'on a fait antérieurement, mais, puisque nous sommes toujours en plein travail, d'y inclure aussi les derniers résultats de nos recherches qui ne sont pas encore publiés, afin de mettre au courant les participants du colloque de nos intérêts actuels. D'une grande importance, aussi, est l'inclusion dans mes exposés de quelques résultats de nombreuses investigations expérimentales faites, ici, dans les laboratoires Français, investigations qui ont été inspirées par nos théories antérieures, et qui heureusement les ont bien vérifiées, telles les expériences faites par le Professeur Quentin, le Docteur de Billy et leur équipe à l'Université Paris VII sur la pénétration d'objets élastiques par l'ultrason de haute fréquence ; et les investigations sur les résonances et les ondes de surface sur des objets élastiques faites par le Professeur Ripoche, le Docteur Maze et leurs collègues à l'Université du Havre, soutenus théoriquement par les Docteurs Derem et Rousselot du Laboratoire Central des Télécommunications à Vélizy-Villacoublay ; plus récemment aussi les expériences des Docteurs Gazanhes et Sessarego au CNRS de Marseille, et autres recherches.

Nous sommes profondément satisfaits que nos suggestions théoriques aient causé, si je peux dire ainsi, autant d'échos en France. J'espère tout spécialement qu'à partir de nos études sur les résonances, et les expériences associées, de nouvelles méthodes et applications vont pouvoir être développées dans le domaine acoustique.

Herbert Überall

Paris, le 21 Novembre 1984

Preface

Professor Quentin and the organizers of this colloquium have honored me by their request that I present the first scientific talk in this Colloquium on the Scattering of Ultrasonic Waves, as well as the opening talks of each session of the colloquium. I greatly appreciate this honor which permits me to present, in the surroundings of the venerable Ecole Normale Supérieure of the University of Paris, to an assembly of renowed scientists and of students aspiring to such a status also, the fruits of twenty years' work on acoustic problems which I have been able to carry out with my collaborators and students at the Department of Physics of the Catholic University of America in Washington DC, as well as with numerous scientists in Government laboratories of the Washington area, with whom I had the good fortune to be able to collaborate. In most of my lectures, I took pains to present not only a résumé of what was done by us previously, but, since we always keep on working, to include also the latest results of our studies which have not yet been published, in order to keep the participants of the colloquium informed of our present interests. Of great importance, additionally, is the inclusion in my talks of some of the results of numerous experimental studies that were carried out here, in French laboratories, studies which were induced by our earlier theories, and which they fortunately have well verified, such as the experiments of Professor Quentin, Dr. de Billy and their team at the University of Paris VII on the penetration of elastic objects by high-frequency ultrasound; and the experiments on resonances and surface waves on elastic objects carried out by Professor Ripoche, Dr. Maze and their colleagues at the University of Le Havre, supported theoretically by Drs. Derem and Rousselot of the Laboratoire Central des Télécommunications of Vélizy-Villacoublay, and recently also the experiments of Drs. Gazanhes and Sessarego at the CNRS at Marseilles, and those of other scientists. We are profoundly satisfied

that our theoretical suggestions have caused, if I may say so, such an echo in France. I specially hope that out of our studies on resonances, and of the related experiments, new methods and applications will be developed in the field of Acoustics.

Herbert Überall
Paris
November 21, 1984

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I wish to dedicate this volume to my present and former students, whose enthusiasm has helped me in carrying out, together with them, the described research work.

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H. Überall

Paris, December 12, 1984

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DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

Université Paris VII

4 - 7 Décembre 1984

L'ACOUSTIQUE DES FAISCEAUX BORNES

par H. ÜBERALL

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RESUME : On obtient l'expression du faisceau acoustique borné le plus général, c'est-à-dire celle d'un faisceau fini dans deux dimensions, émergeant d'une ouverture collimatrice de forme arbitraire, et possédant un profil arbitraire d'amplitude. On étudie la divergence géométrique d'un tel faisceau pour deux cas de profil d'amplitude : un profil gaussien et un profil plan-rectangulaire. Dans chaque cas, on obtient la divergence géométrique comme fonctio du rayon correspondant à la dimension de l'ouverture collimatrice et de trois paramètres différents : longueur d'onde, distance axiale du plan de l'ouverture, et distance radiale de l'axe du faisceau. Comme application de cette théorie générale, on obtient l'amplitude de diffusion d'un faisceau borné par une sphère rigide d'une façon exacte pour les deux profils mentionnés du faisceau; et on fait une comparaison entre les résultats concernant la section efficace, et ceux obtenus par la méthode de Kirchhoff. Mention est faite d'autres représentations d'un faisceau borné rencontrées dans la littérature.

SUMMARY : We derive the expression for the most general bounded acoustic beam, i.e., for a beam finite in two dimensions, emerging from an arbitrarily shaped collimating aperture at an arbitrary angle and with an arbitrary amplitude profile. We investigate the spreading of such a beam for two amplitude profiles : a Gaussian profile and a flat-topped square profile. In both cases the spreading is determined as a function of the ratios between the collimating aperture sizes and three parameters : wavelength, axial distance from the aperture plane, and radial distance from the beam axis. As a specific application of this general theory, we evaluate the scattering of a finite beam from a rigid sphere in an exact fashion for the two mentioned amplitude profiles of the finite beam, and compare results for the bistatic cross sections to those obtained using

the Kirchhoff approximation. We also mention other representations for finite beams encountered in the literature.

1. INTRODUCTION :

Des faisceaux bornés ont été étudiés théoriquement et expérimentalement. Schoch [1,2] a étudié des effets de déplacement apparents d'un faisceau borné réfléchi par un solide ; d'autres études sur ce sujet ont été faites par Mayer et al [3,4] et Breazeale et al [5,6]. Les expériences de Neubauer [7], et les théories de Tamir et Bertoni [8,9] ont clarifié le rôle joué par l'onde de Rayleigh dans ces phénomènes. Plus récemment, une autre théorie a été faite par Claeys et Leroy [10] concernant l'interaction d'un faisceau borné avec la surface plane d'un solide ; la réflexion d'un faisceau borné par une surface rugueuse (périodique) a été étudiée également [11].

Dans tous ces travaux, on a considéré des faisceaux acoustiques bornés dans une dimension seulement, ce qui était suffisant pour la description théorique des expériences mentionnées. Pourtant, l'extension de la théorie au cas d'un faisceau borné dans deux dimensions ne paraît pas très difficile ; les descriptions de tels faisceaux ont été donnés par Gaunaurd et Überall [12] et, indépendamment, par Alais et Hennion [13]. Toutes les théories représentent les faisceaux bornés par une superposition continue d'ondes planes, dans le sens de Fourier, se propageant dans des directions diverses. La seule exception est faite par Claeys et Leroy [10] qui représentent le faisceau borné (à une dimension) par une superposition discrète d'ondes planes inhomogènes [14,15], se propageant dans la même direction. Cette représentation a l'avantage d'une description analytique du faisceau réfléchi, d'une façon analogue à la méthode de Prony [16], tandis que la méthode de Fourier nous fournit des expressions intégrales.

Dans ce qui suit, on développe la théorie d'un faisceau acoustique borné dans deux dimensions basée sur la méthode de Fourier, et on obtient la divergence géométrique d'un faisceau sortant d'une ouverture collimatrice. Comme application de notre représentation du faisceau borné, on considère la diffusion d'un tel faisceau par une sphère ; les résultats exacts obtenus ainsi pour la section efficace de diffusion peuvent être comparés avec ceux de la diffusion d'une onde plane par la sphère, et ceux de la diffusion du faisceau analysée par la méthode de Kirchhoff.

2. FAISCEAUX BORNES DANS DEUX DIMENSIONS.

Un faisceau infini,

$$p_\infty(\vec{r}) = e^{i\vec{k}_o\cdot\vec{r}}, \tag{2.1}$$

incident de $-\infty$ sur un plan $z = -L$, a pour expression

$$p_\infty(\vec{r})\Big|_{z=-L} = P_o(\vec{\rho})\, e^{i\vec{\kappa}_o\cdot\vec{\rho}}\, e^{-ik_{oz}L} \tag{2.2}$$

dans une ouverture de ce plan, où

$$\vec{\rho} = (x, y), \quad \vec{r} = (\vec{\rho}, z), \quad \vec{r}_L = (\vec{\rho}, -L), \tag{2.3a}$$

$$\vec{k}_o = (\vec{\kappa}_o, \; k_{oz}). \tag{2.3b}$$

On a introduit ici le profil $P_o(\vec{\rho})$ de l'amplitude du faisceau à travers l'ouverture.

Le faisceau le plus général peut maintenant être représenté par l'expression de Fourier,

$$p(\vec{r}) = \frac{1}{(2\pi)^2} \iint \Psi(\vec{\kappa})\, e^{i\vec{k}\cdot\vec{r}}\, d^2\kappa, \tag{2.4}$$

avec la relation de dispersion

$$k_z = \pm\,(k^2 - \vec{\kappa}^2)^{1/2}, \qquad z' \equiv z + L \gtrless 0 \;\; (\vec{\kappa}^2 < k^2) \tag{2.5a}$$

$$k_z = \pm\, i\,(\vec{\kappa}^2 - k^2)^{1/2}, \qquad z' \equiv z + L \gtrless 0 \;\; (\vec{\kappa}^2 > k^2), \tag{2.5b}$$

les signes étant choisis pour assurer la convergence quand $|z'| \to \infty$

L'expression (2.4) est une solution de l'équation de Helmholtz, se réduisant correctement à l'onde dans l'ouverture, Eq.(2.2), si $\Psi(\vec{\varkappa})$ est la transformée de Fourier de cette onde,

$$\Psi(\vec{\varkappa}) = \int P_o(\vec{\rho})\, e^{i[(\vec{\varkappa}_o - \vec{\varkappa})\cdot\vec{\rho} - (k_{oz} - k_z)L]}\, d^2\rho. \tag{2.6}$$

Les équations (2.4) - (2.6) précisent entièrement le faisceau borné dans deux dimensions, si on considère l'argument de Claeys et Leroy [10] que pour des faisceaux larges, $\partial P_o/\partial z$ peut être présumé comme proportionnelle à P_o. Dans tous les travaux antérieurs, les expressions de Fourier comme (2.4), ou d'une superposition analogue d'ondes inhomogènes [10] étaient considérées dans une dimension seulement, et représentaient donc un faisceau borné dans une dimension, c'est-à-dire un faisceau en forme d'un ruban plan.

Pour un faisceau à symétrie de révolution, $P_o = P_o(\rho)$, se propageant parallèlement à l'axe z ($\vec{\varkappa}_o = 0$), l'équation (2.4) se simplifie en

$$p(\vec{r}) = e^{-ikL}\int_0^\infty P_o(\rho')\rho'd\rho' \int_0^\infty J_o(\varkappa\rho)\, J_o(\varkappa\rho')\, e^{ik_z z'}\varkappa d\varkappa; \tag{2.7}$$

on peut alors considérer les cas spéciaux de

$$P_o(\rho) = H(R_o - \rho), \tag{2.8a}$$

$$p(\vec{r}) = R_o e^{-ikL}\int_0^\infty J_1(\varkappa R_o)\, J_o(\varkappa\rho)\, e^{ik_z z'} d\varkappa \tag{2.8b}$$

d'un faisceau de profil rectangulaire de rayon R_o, H étant la fonction de Heaviside, ou

$$P_o(\rho) = e^{-\rho^2/2\sigma^2}, \tag{2.9a}$$

$$p(\vec{r}) = \sigma^2 e^{-ikL}\int_0^\infty e^{-\varkappa^2\sigma^2/2}\, J_o(\varkappa\rho)\, e^{ik_z z'}\varkappa d\varkappa \tag{2.9b}$$

d'un faisceau de profil Gaussien.

3. DIVERGENCE GEOMETRIQUE DU FAISCEAU.

Se propageant en dehors de l'ouverture, le faisceau diverge à cause de la diffraction. Les intégrales des équations (2.8b) ou (2.9b) peuvent alors être évaluées [12] par la méthode de col, ou par intégration partielle.

La figure 3.1 présente en variables sans dimension, la divergence géométriqu d'un faisceau rectangulaire à l'ouverture ($v = 0$), à la fréquence $s = kR_o = 8$. La figur 3.2 donne la divergence géométrique d'un faisceau gaussien à la fréquence $k\sigma = 8$.

4. APPLICATION : DIFFUSION D'UN FAISCEAU PAR UNE SPHERE [12].

Le champ total d'une onde plane en présence d'une sphère de rayon a est la somme de l'onde incidente p_{inc} et de l'onde diffusée p_{sc},

$$p(\vec{r},\vec{k}) = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} i^{\ell} \left[j_\ell(kr) + C_\ell(ka) h_\ell^{(1)}(kr) \right] Y_{\ell m}^{*}(\hat{k}) Y_{\ell m}(\hat{r}), \quad (4.1a$$

l'amplitude diffusée $C_\ell(ka)$ étant, dans le cas d'une sphère rigide :

$$C_\ell(ka) = - j_\ell'(ka) / h_\ell^{(1)\prime}(ka). \quad (4.1b$$

La section efficace bistatique (ou distribution angulaire) de diffusion est donnée par

$$\frac{d\sigma}{d\Omega} = \lim_{r \to \infty} r^2 \left| \frac{p_{sc}}{p_{inc}} \right|^2, \quad (4.2)$$

qui devient en champ lointain

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{k} \right)^2 \left| \sum_{\ell=0}^{\infty} C_\ell (2\ell+1) P_\ell(\cos\theta) \right|^2. \quad (4.3$$

Un faisceau borné a pour champ diffusé

$$p_{sc}(\vec{r}) = \frac{1}{\pi}\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} i^{\ell} C_{\ell}\, h_{\ell}^{(1)}(kr)\, Y_{\ell m}(\hat{r}) \int \Psi(\vec{\varkappa})\, Y_{\ell m}^{*}(\hat{k})\, d^{2}\varkappa, \tag{4.4}$$

ce qui donne en champ lointain dans le cas d'un faisceau circulaire :

$$\frac{4}{a^{2}}\frac{d\sigma}{d\theta} = \left| \frac{2}{ka}\sum_{\ell=0}^{\infty} (2\ell+1)\, C_{\ell}(ka)\, I_{\ell}\, P_{\ell}(\cos\theta) \right|^{2} \tag{4.5a}$$

où

$$I_{\ell} = \frac{1}{2\pi}\int_{0}^{\infty} \Psi(\varkappa)\, P_{\ell}(\cos\theta_{k})\, \varkappa\, d\varkappa, \tag{4.5b}$$

$\varkappa = k \sin \theta_k$. Pour un faisceau de profil rectangulaire :

$$I_{\ell} = 1 - f\, P_{\ell}(f)\, e^{ikL(1-f)/f}, \tag{4.6a}$$

$$f = \left\{ 1 + (R_o/L)^{2} \right\}^{-1/2}, \tag{4.6b}$$

en utilisant la méthode du col.

La figure 4.1 présente, pour une sphère rigide, $d\sigma/d\theta$ dans le cas du faisceau rectangulaire, pour L = 1 et ka = 3 (à gauche) ou ka = 5 (à droite), pour le rayon du faisceau r = a où r = 0.866a (et également r = 400 a , en haut). Pour comparaison, la figure 4.2 montre les mêmes résultats obtenus par la méthode de Kirchoff [12,17], sauf pour le cas r > a qui coïncide avec le cas r = a.

5. CONCLUSION

Les calculs précédents montrent que :

(a) un faisceau borné dans deux dimensions peut être représenté par une intégrale bidimensionnelle de Fourier qui

(b) permet un calcul de la divergence géométrique du faisceau émergeant d'une ouverture.

(c) rend possible l'évaluation de la diffusion du faisceau borné par une cible.

(d) admet l'analyse de l'interaction du faisceau borné en deux dimensions avec des structures telles que le dioptre ou la plaque, plan ou courbé, ainsi que le calcul, par exemple, d'un faisceau dans un milieu inhomogène. Une partie des résultats rapportés est basée sur des travaux assistés financièrement par l'ONR et le NSWC.

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LEGENDE

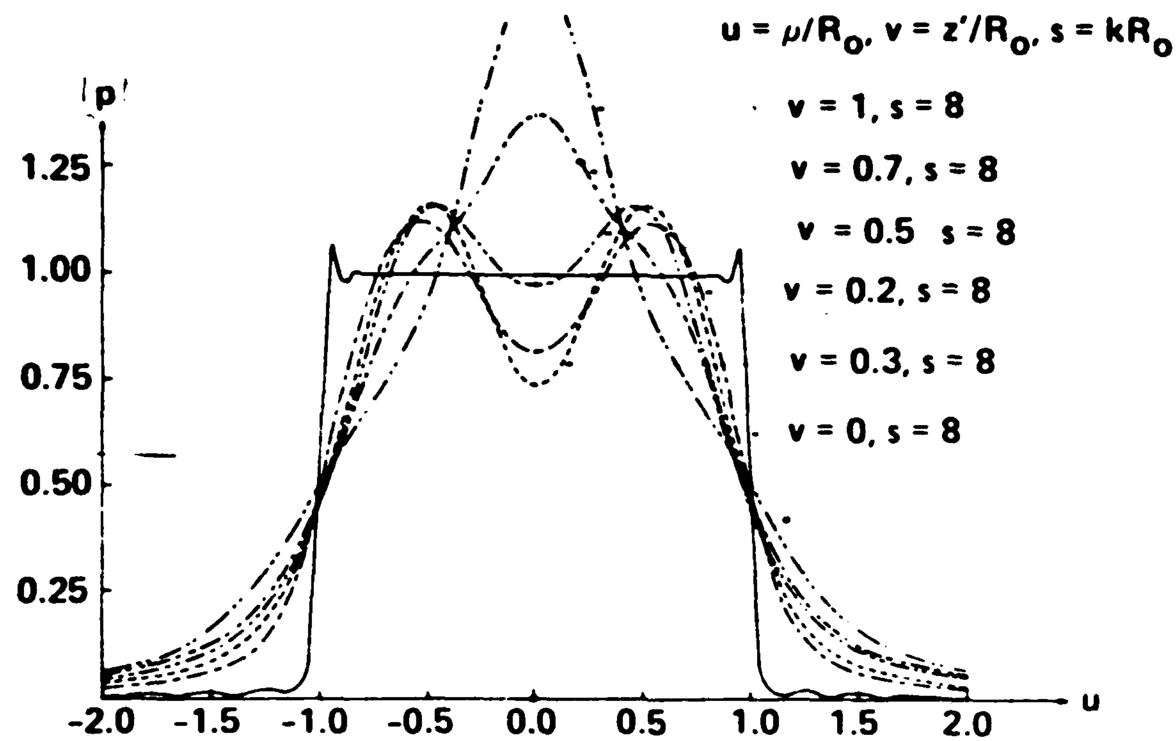
Fig. 3.1. Etendue diffractive d'un faisceau de profil rectangulaire, avec $s = kR_o = 8$.

Fig. 3.2. Etendue diffractive d'un faisceau de profil Gaussien ($x = \rho/\sigma$, $y = z'/\sigma$), avec $w = k\sigma = 8$.

Fig. 4.1. Distributions angulaires de la diffusion d'un faisceau rond, avec profil rectangulaire, d'une sphère rigide.

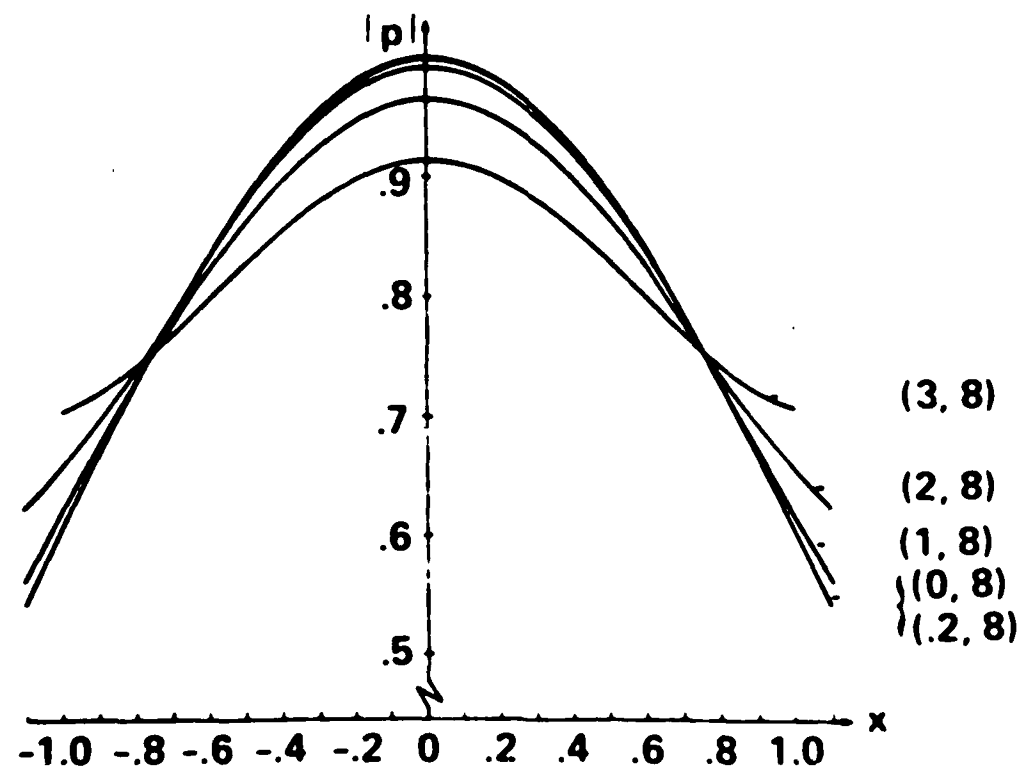
Fig. 4.2 Résultats comparables à ceux de la figure 3, obtenus par l'approximation de Kirchhoff.

Fig. 3.1



$u = \rho/R_o,\ v = z'/R_o,\ s = kR_o$
v = 1, s = 8
v = 0.7, s = 8
v = 0.5 s = 8
v = 0.2, s = 8
v = 0.3, s = 8
v = 0, s = 8
|p|
1.25
1.00
0.75
0.50
0.25
-2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0
u

Fig. 3.2



|p|
.9
.8
.7
.6
.5
(3, 8)
(2, 8)
(1, 8)
(0, 8)
(.2, 8)
-1.0 -.8 -.6 -.4 -.2 0 .2 .4 .6 .8 1.0
x

Fig. 4.1

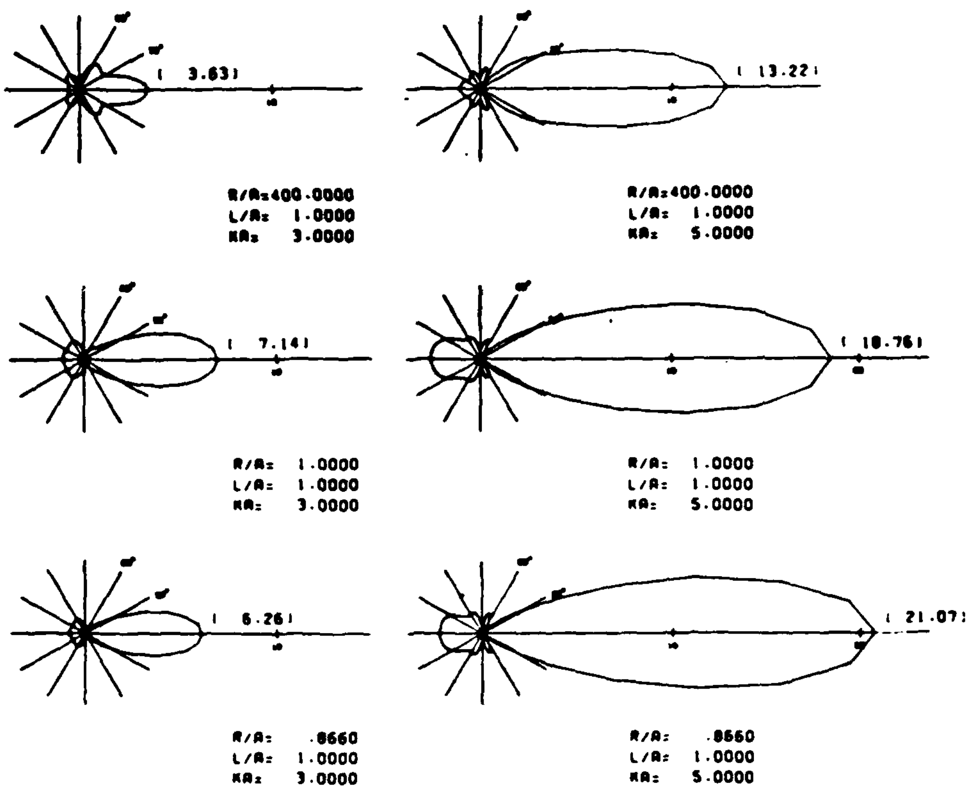
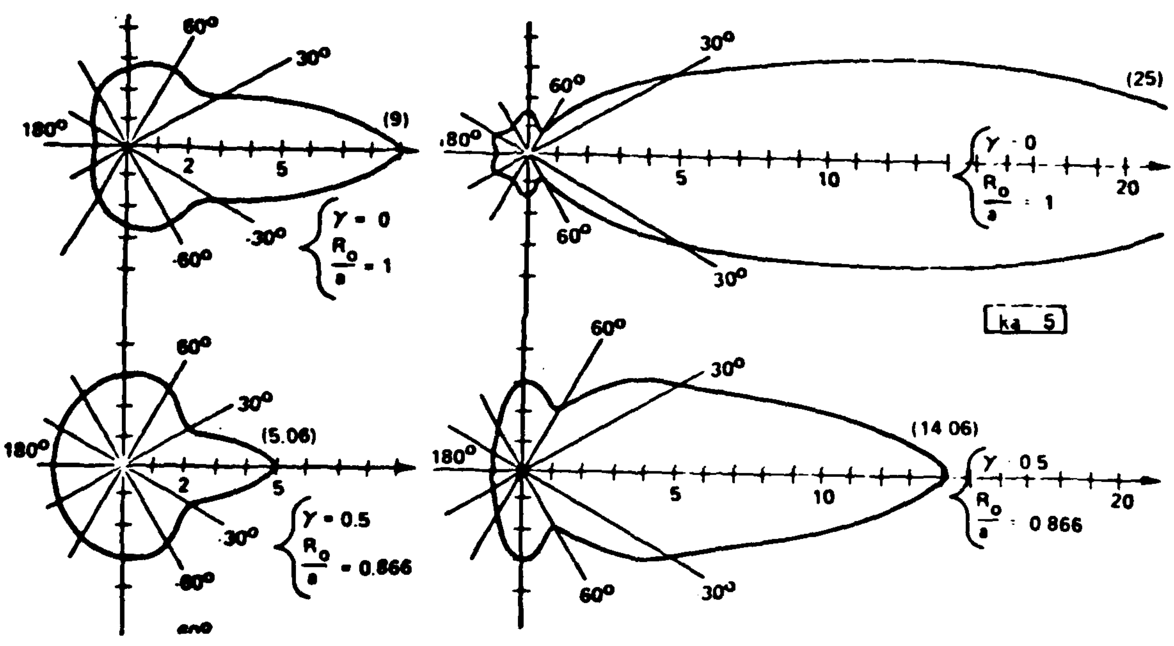

(3.63)
(13.22)
R/A=400.0000
L/A= 1.0000
KA= 3.0000
R/A=400.0000
L/A= 1.0000
KA= 5.0000
(7.14)
(18.76)
R/A= 1.0000
L/A= 1.0000
KA= 3.0000
R/A= 1.0000
L/A= 1.0000
KA= 5.0000
(6.26)
(21.07)
R/A= .8660
L/A= 1.0000
KA= 3.0000
R/A= .8660
L/A= 1.0000
KA= 5.0000

Fig. 4.2


60°
30°
180°
(9)
-30°
-60°
γ = 0
$\frac{R_o}{a}$ = 1
(25)
ka 5
60°
30°
(5.06)
γ = 0.5
$\frac{R_o}{a}$ = 0.866
(14 06)
γ 0 5
$\frac{R_o}{a}$ 0 866

DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

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SCATTERING FROM FLUID AND ELASTIC LAYERS

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SUMMARY : In this report, we describe a complete resonance theory for the acoustic transmission and reflection coefficients of an elastic plate imbedded in a fluid medium, including effects of plate viscosity. The purpose of this formulation is to provide a direct means for determining the material parameters of the plate from the measured acoustic resonances of the Rayleigh and Lamb waves in the plate (i.e., their positions in frequency or angle, their widths and their heigths) which are given in our formalism by explicit analytic expressions that depend on the material parameters. Viscosity is seen to manifest itself in a decrease of the resonance heights (especially for the narrow shear-type resonances) and in a broadening and frequency dependence of their widths, which may be used to determine the frequency-dependent loss factor of the plate. This approach then solves the inverse scattering problem for the case of a plate. We also consider the special case of a fluid layer imbedded in another fluid. In addition, a layered ocean floor is modeled by a sediment layer on top of a denser substratum, and overlaid by the water column. It is shown for this case again that resonances in the acoustic reflection coefficient are very prominent and can be used to determine the properties of both sediment layer and substratum.

RESUME : Dans cet article, nous présentons une théorie complète des résonances dans les coefficients de transmission et de réflexion acoustique par une plaque élastique immergée dans un milieu fluide, en tenant compte des effets dus à la viscosité de la plaque. Le but de ce formalisme est de fournir un moyen direct de détermination des paramètres du matériau constituant la plaque,par la mesure des résonances des ondes de Rayleigh et de Lamb (c'est-à-dire, par leur position en fréquence ou en angle, leurs largeurs et leurs hauteurs). Ces quantités sont données dans notre formalisme

par des expressions analytiques explicites qui contiennent les paramètres du matériau. On trouve que la viscosité se manifeste par une décroissance des hauteurs des résonances (particulièrement pour les résonances étroites du type transversal), et par un élargissement et une dépendance en fréquence de leurs largeurs. Cette méthode permet de résoudre alors le problème de diffusion inverse pour le cas d'une plaque. Nous considérons également le cas spécial d'une couche fluide immergée dans un autre fluide. En outre, nous représentons les couches du fond de l'océan par une couche de sédiment posée sur un fond plus dense, et couverte par l'eau de l'océan. Nous montrons que, pour ce cas également, les résonances dans le coefficient de réflexion acoustique apparaissent de façon très visible, et peuvent être utilisées pour la détermination des propriétés de la couche de sédiment et du fond.

1. INTRODUCTION.

A comprehensive theoretical and experimental investigation by Schoch [1] has shown that the process of reflection and transmission of an acoustic wave, incident through a fluid on an elastic plate imbedded in that fluid, is capable of exciting elastic Rayleigh and Lamb-type waves [2] in the plate. Such an excitation takes place when the trace of the phase velocity of the incident wave, as it moves along the plate, coincides [3] with the phase velocity of the plate waves ; it manifests itself in the appearance of resonances[4] in the acoustic reflection (R) and transmission coefficient (T) of the plate, either as the frequency f is varied at a fixed angle of incidence θ from the normal, or as θ is varied at a fixed frequency.

Mathematically, such resonances in R or T come about by a vanishing of the common denominator in the fractional analytic expressions for these two quantities. Zeros of this denominator, i.e., poles in R and T, do not occur for real values of both f and θ ; they occur, e.g., at complex values of f if θ is kept real (physical), or vice versa. Since experimentally both f and θ are real, measurements with variation of f will carry us successively past the poles which are located somewhat off the real f-axis, thus leading to finite heights and widths of the observed resonance peaks.

The resonance features in R and T can be studied by isolating their pole contributions, that is, by replacing the denominator by its first term in a Taylor - series expansion about each of the poles. This was done by Fiorito, Madigosky and Überall for the cases of a fluid layer in another fluid, the layer being either lossless [4] or lossy [5], as well as for an elastic plate imbedded in a fluid, with the plate being lossless [6] or lossy [7]. It is possible, however, to go beyond this approach, and to

represent R and T by an exact series representation as a sum of resonance terms in the sense of Mittag-Leffler [8] ; this is also akin to the theory of resonances in nuclear reactions, known as the Breit-Wigner theory [9]. Such an exact representation is discussed here, as given recently by Fiorito, Madigosky and Überall [10] for the case of a fluid layer. The merit of such an exact resonance décomposition rests in the fact that it admits a description of the resonances even for small impedance contrasts between the layer and the ambient fluid.

The mentioned investigations [4-7,10] have provided analytical expressions for the resonance positions (in f or θ), their heights and their widths. These expressions depend on the materials parameters of the plate, and the measurement of these quantities thus furnishes a direct means for the determination of the material parameters by remote acoustic sensing. In a way, such a procedure then corresponds to a solution of the "inverse scattering problem", where the properties of the scatterer are determined from the form of the returned echo.

An experimental study of plate resonances has recently been carried out by Izbicki, Maze, and Ripoche [11-13], showing in exact detail the backscattering resonances in the reflection coefficient for aluminum plates immersed in water as plotted vs. f, and featuring their positions in f, their heights and widths.

Fluid layers have also been used to model sediment layers of the ocean bottom, lying on top of a substratum which was also treated as a fluid [14]. Calculated reflection coefficients for a layered ocean bottom show very prominent resonance features [15]. These are explained in the mentioned model, which proceeds with a detailed analysis to show what types of measurements have to be carried out on these resonance features in order to completely determine the geometrical and material properties of layer and substratum. Finally, a way of analyzing the layer resonances via the use of long sound pulses, which are employed in order to cause a ringing of the layer resonances one at a time, has also been pointed out [16].

2. THEORY OF LAYER RESONANCES.

The exact expression for the transmission and reflection coefficient of an elastic layer as calculated by Schoch [1] is first rewritten in a convenient form :

$$T = i\tau \left(\frac{1}{C_s - i\tau} + \frac{1}{C_\alpha + i\tau} \right) , \qquad (2.1a)$$

$$R = \frac{C_s C_a - \tau^2}{C_s + C_a}\left(\frac{1}{C_s - i\tau} + \frac{1}{C_a + i\tau}\right), \tag{2.1b}$$

with
$$\tau = \frac{\rho}{\rho_d}\,\frac{(n_d^2 - \sin^2\theta)^{1/2}}{\cos\theta}, \tag{2.2a}$$

and $n_d = c/c_d$, where d is the layer thickness, and c (c_d, c_t) and ρ (ρ_d) are the acoustic(dilatational d, and transverse t) wave speeds and densities in the external (or layer) media, respectively. The quantities C_s and C_a depend [6] on these parameters or combinations these of, such as on

$$\delta = \frac{1}{2}\, x\, (n_d^2 - \sin^2\theta)^{1/2} \tag{2.2b}$$

where $x = 2\pi f\, d/c$. Viscosity is described by complex wave velocities c_d^*, c_t^*. For the case of a fluid layer ($c_t = 0$), one has $C_s = \cot\delta$, $C_a = \tan\delta$.

A taylor expansion of the denominators of Eqs. (2.1) leads to the series of resonance terms (of Breit-Wigner form)

$$T \equiv T_s + T_a = \sum_{m_s} T_{m_s} + \sum_{m_a} T_{m_a}, \tag{2.3a}$$

$$R \equiv R_s + R_a = \sum_{m_s} R_{m_s} + \sum_{m_a} R_{m_a}, \tag{2.3b}$$

where

$$T_{m_{s,a}} \cong \mp \frac{(i/2)\Gamma_{m_{s,a}}}{x - x_{m_{s,a}} + (i/2)(\Gamma_{m_{s,a}} + \Delta\Gamma_{m_{s,a}})}, \tag{2.4a}$$

$$R_{m_{s,a}} \cong \frac{x - x_{m_{s,a}} + (i/2)\Delta\Gamma_{m_{s,a}}}{x - x_{m_{s,a}} + (i/2)(\Gamma_{m_{s,a}} + \Delta\Gamma_{m_{s,a}})}. \tag{2.4b}$$

The (normalized) resonance frequencies $x_{m_{s,a}}$ are found as the solutions of the lossless free-plate characteristic equations

$$d_{s,a}(x_{m_{s,a}}) = 0, \tag{2.5}$$

and the resonance widths $\Gamma_{m_{s,a}}$ as well as their loss broadening $\Delta\Gamma_{m_{s,a}}$ are given [6,7] as analytic expressions in terms of the material parameters of the plate. As mentioned, this leads to the possibility of determining these parameters from a measurement of the resonance features.

The derivation of Eqs. (2.3) - (2.5) was based on the assumption of a large impedance contrast between fluid and plate, i.e., $\tau \ll 1$. A Mittag-Leffler type meromorphic series expansion gives, e.g., for a lossless fluid layer :

$$T = \frac{\tau}{\sigma(1-\tau^2)}\left\{\sum_{m_s=-\infty}^{\infty} \frac{-\frac{1}{2}i\Gamma}{x - x_{m_s} + \frac{1}{2}i\Gamma} + \sum_{m_a=-\infty}^{\infty} \frac{\frac{1}{2}i\Gamma}{x - x_{m_a} + \frac{1}{2}i\Gamma}\right\} \tag{2.6}$$

where $\sigma = \tanh^{-1}\tau$,

$$x_{m_s} = (2m_s + 1)\pi\,(n_d^2 - \sin^2\theta)^{-1/2}, \tag{2.7a}$$

$$x_{m_a} = 2m_a\pi\,(n_d^2 - \sin^2\theta)^{-1/2}, \tag{2.7b}$$

$$\frac{1}{2}\Gamma = \frac{2\sigma}{(n_d^2 - \sin^2\theta)^{1/2}} . \tag{2.7c}$$

Equation (2.6) is an exact representation for T, valid for all $\tau < 1$. A corresponding expression can be found for $\tau > 1$ [10].

3. NUMERICAL RESULTS.

Figure 3.1 shows the resonances in $|T|^2$ vs. f at $\theta = 19°$, for a one-inch thick absorptive plexiglas plate in silicone oil, representing plots of the individual resonance terms in the present formalism [7]. The classification in terms of antisymmetric (a) and symmetric (s) plate waves, as well as their dilatational (d) and transverse (t) content follows Brekhovskikh [2].

Figure 3.2 then shows the coherent sum of the terms of the present FMU resonance theory, compared to the result of the exact expression, Eq. (2.1a). The agreement is adequate at the prevailing impedance contrast ($\tau = 0.211$ at $\theta = 0°$), and it also shows that the representation by individual resonances of the FMU theory provides a means of resolving cases of interfering resonances in the coherent sum, such as at $f \simeq 60$, 105 and 155 kHz.

4. OCEAN FLOOR RESONANCES.

Calculated reflection coefficients for models of a layered ocean floor often contain prominent resonances [15]. Figure 4.1 shows a two-dimensional plot of $|R|^2$ for two (fluid) sediment layers on top of a fluid substratum, plotted vs. both f and θ [14]. In this reference, a detailed analysis is given how to determine the bottom structure from measurements on the resonances a large redundancy of measurements was found to apply. In addition, the ringing of layer resonances as manifested by a tail in long, specularly reflected wavetrains (Fig. 4.2) has been suggested [16] as a means of measuring the resonance properties.

5. CONCLUSION.

The analysis of resonances in the acoustic reflection from layers and plates has shown that reflection and transmission coefficients can be represented as a sum of resonance terms, of Breit-Wigner form familiar from nuclear physics, either in the "resonance approximation" or in an exact meromorphic series. The resonance positions (in frequency, or in angle of incidence), heights, and widths depend on the material parameters, and their measurement may thus serve to determine these parameters by remote acoustic sensing. An explicit demonstration of such a procedure was provided [14,16] for the case of sediment layers on the ocean floor.

Portions of the results reported here are based on work supported by the ONR and the NSWC.

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FIGURE CAPTIONS

Fig. 3.1. Individual resonances of $|T|^2$ from the resonance theory, for a 1-inch plexiglas plate in silicone oil at $\theta = 19°$.

Fig. 3.2. Coherent sum of resonances in $|T|^2$ for a 1 - inch plexiglas plate in silicone oil at $\theta = 19°$, compared with exact result.

Fig. 4.1. $|R|^2$ for two ocean - floor sediment layers on top of a substratum, plotted vs. frequency - thickness and incident angle.

Fig. 4.2. Reflected signal vs. time, for incident wave train with carrier frequency equalling a resonance frequency of an ocean floor sediment layer. Final transient is due to ringing of layer resonance.

Fig. 3.1

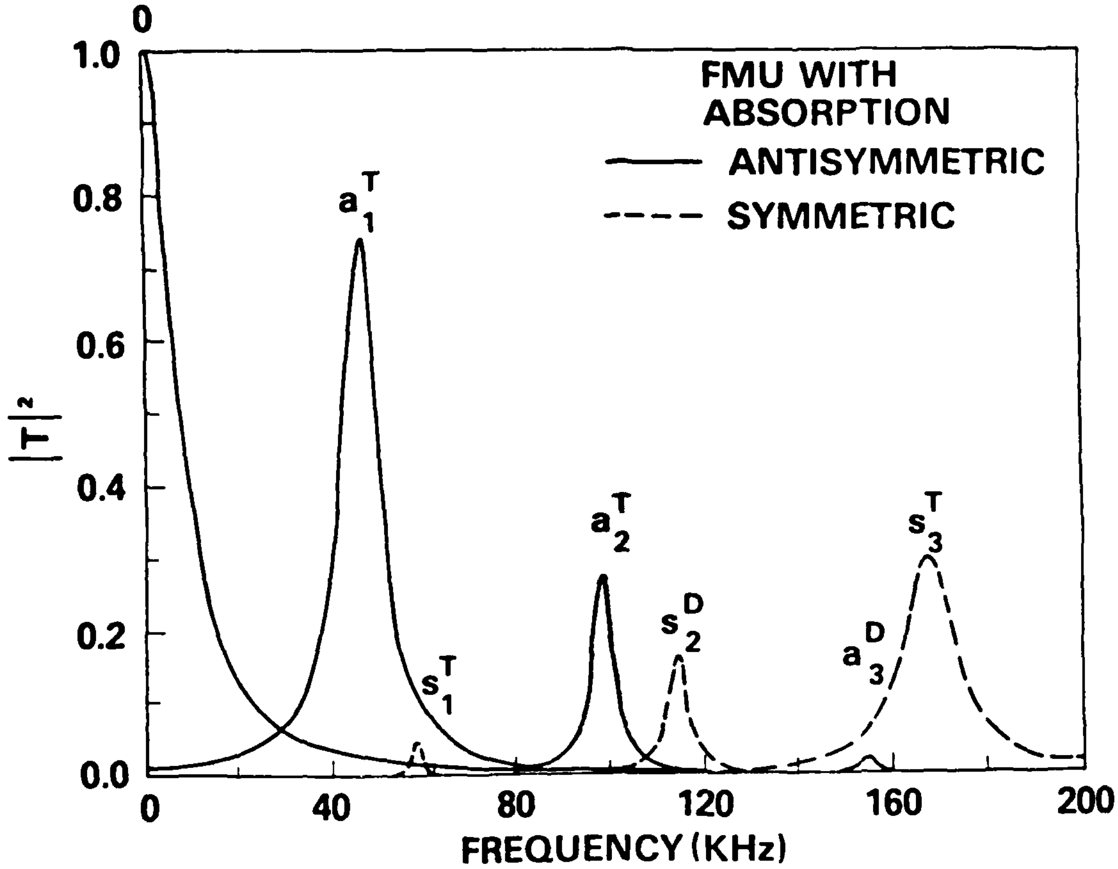

0
1.0
0.8
0.6
0.4
0.2
0.0
$|T|^2$
FMU WITH ABSORPTION
ANTISYMMETRIC
SYMMETRIC
a_1^T
s_1^T
a_2^T
s_2^D
a_3^D
s_3^T
0
40
80
120
160
200
FREQUENCY (KHz)

Fig. 3.2

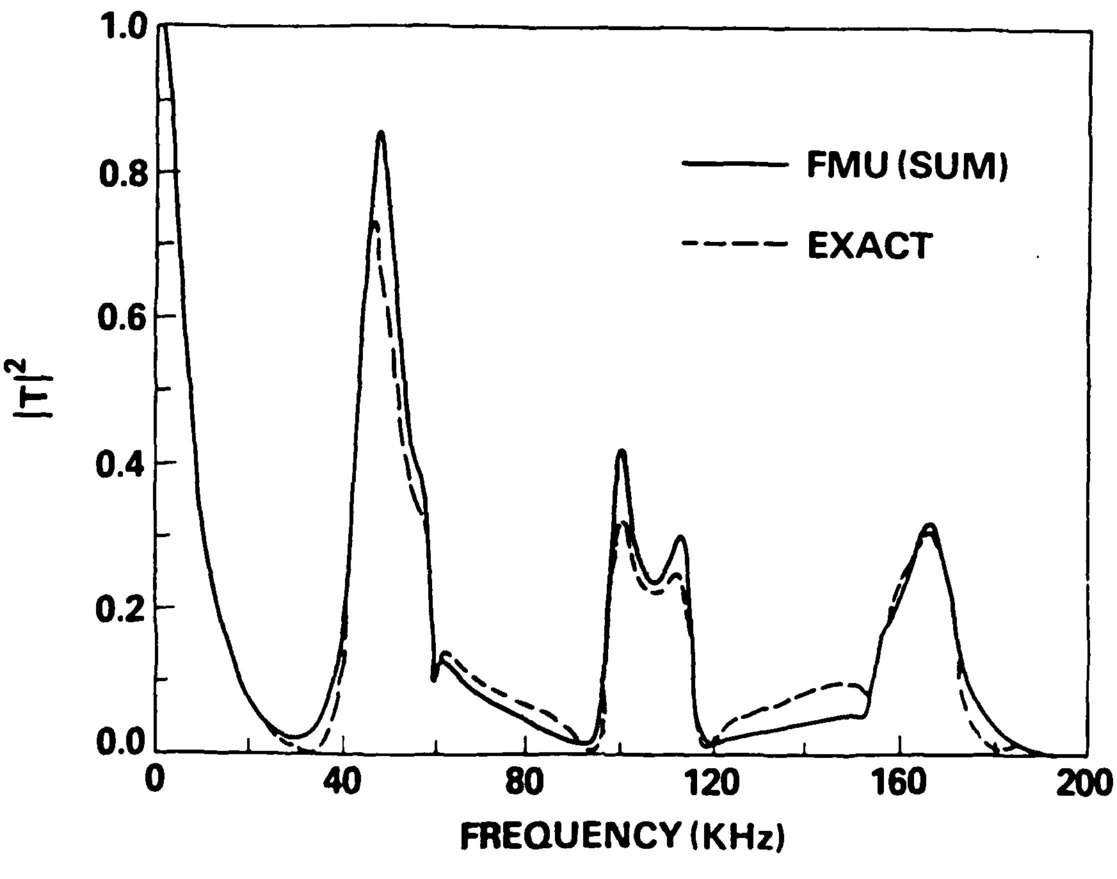

1.0
0.8
0.6
0.4
0.2
0.0
$|T|^2$
FMU (SUM)
EXACT
0
40
80
120
160
200
FREQUENCY (KHz)

Fig. 4.1

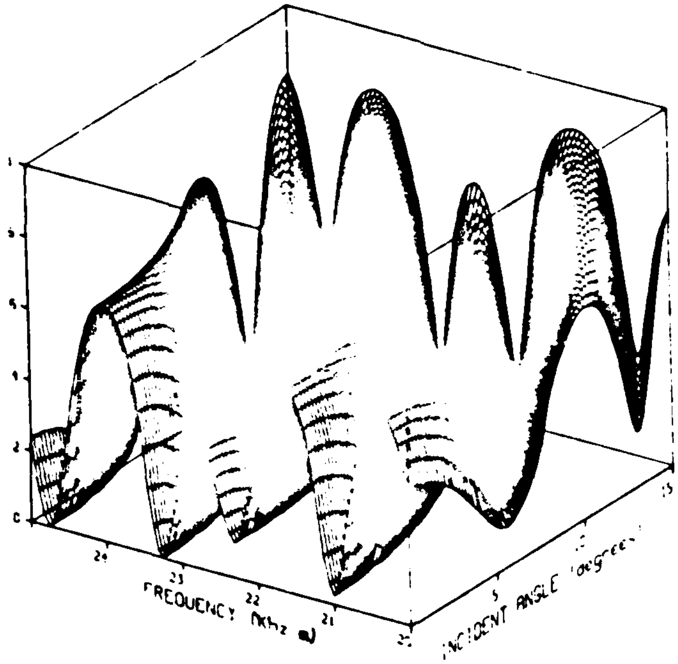
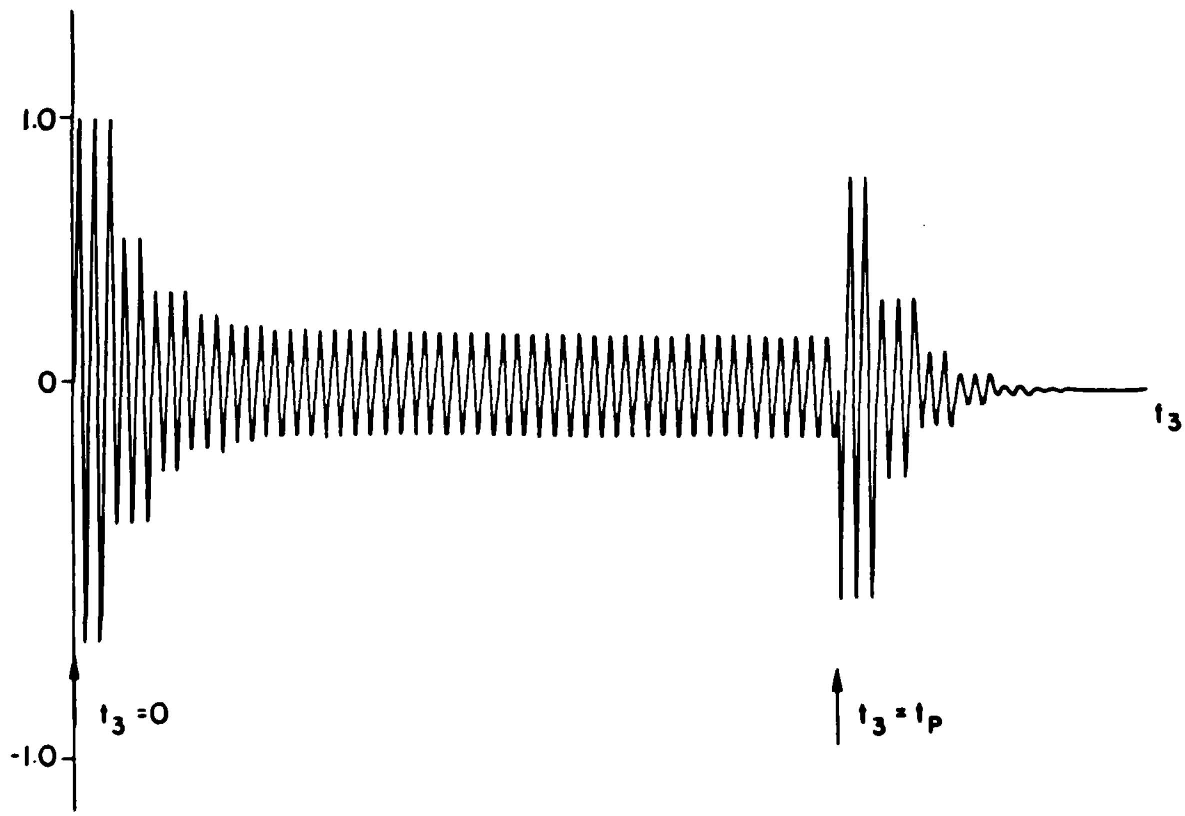


Fig. 4.2



DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

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ACOUSTIC SCATTERING FROM ELASTIC CYLINDERS AND SPHERES : SURFACE WAVES (WATSON TRANSFORM) AND TRANSMITTED WAVES.

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SUMMARY : The classical normal-mode series of acoustic scattering from solid elastic cylinders and spheres is reformulated in terms of the S - function as developed in nuclear scattering theory. It is then subjected to the Watson transformation, which permits an evaluation of the scattering amplitude at its poles ("Regge poles") and saddle points in the complex mode-number plane. The saddle point contributions are obtained after expanding the amplitude in a Debye series, and correspond to a reflected wave and to transmitted dilatational and shear waves that undergo internal reflections and mode conversions. The theory of these waves was experimentally verified by Quentin et al. The pole residues furnish circumferential (surface, creeping) waves which are of both Franz type (propagating externally), and of elastic type (Rayleigh and Whispering Gallery waves, propagating internally). The theory of these waves was experimentally verified by Ripoche et al.

RESUME : On reformule la série classique des modes normaux, qui décrit la diffusion acoustique par des cylindres et des sphères solides élastiques en l'exprimant par la fonction S, comme elle est développée dans la théorie de la diffusion nucléaire. Elle est alors soumise à la transformation de Watson, ce qui permet une évaluation de l'amplitude de diffusion à ses pôles ("pôles de Regge") et à ses cols dans le plan complexe du numéro de mode. Les contributions des cols sont obtenues après le développement de l'amplitude en "série de Debye", et elles correspondent à une onde réfléchie et à des ondes transmises de type dilatation et cisaillement, soumises à des réflexions internes et des conversions de mode. La théorie de ces ondes a été vérifiée expérimentalement par Quentin et al. Les résidus des pôles fournissent des

ondes circonférentielles (ondes de surface, "Creeping waves", ou du type de Franz, se propageant sur le côté externe de l'interface), ou du type élastique (ondes de Rayleigh et de galerie à écho, se propageant sur le côté interne de l'interface). La théorie de ces ondes a été vérifiée expérimentalement par Ripoche et al.

1. INTRODUCTION.

The first comprehensive theory of sound scattering from elastic cylinders and spheres was presented by Faran [1] in 1951 ; further extensive work on spheres is due to Hickling [2]. In their analysis, the acoustic field is represented as a Rayleigh - type series summed over normal modes labeled by the mode number n, the modes having been obtained by the separation of variables in cylindrical or spherical coordinates, respectively. Although this solution can be numerically evaluated to obtain, e.g., the backscattering cross section as a function of frequency, it does not lend itself to any physical understanding of the scattering process.

Such an understanding was provided, however, by the work of Franz [3,4] on the scattering of electromagnetic waves from perfect conductors, this being the analogue of acoustic scattering from impenetrable (soft or rigid) objects. His application of the Watson transformation [5] provided a separation of the scattered field into two parts ,one being a wave which in the limit of large values of ka (with k the acoustic wave number, a the radius of the cylinder or sphere) corresponds to geometrical reflection from the apex of the object, and the other representing a series of surface waves (called "creeping waves" by Franz) which propagate circumferentially around the scatterer, thus in effect making up the diffraction phenomenon.

This approach was carried further, and applied to the case of penetrable cylinders [6-9] and spheres [10,11]. Here, interior fields are present, and the following separation of the total field can be made : (a) a "geometric" part which in the large-ka limit corresponds to both a specular reflection from the apex, and to rays refracted into the interior of the scatterer, which then re-emerge into the exterior fluid either immediately, or after a series of multiple internal reflections. For an elastic object, mode conversion into rays of shear type may occur during these refractions or internal reflections ; (b) a surface wave part in which circumferential waves propagate around the scatterer both externally (of "Franz type", similar to those for impenetrable objects), and internally (of "Rayleigh" and "Whispering Gallery" type, the former corresponding to the Rayleigh wave in the limit of a flat surface).

Comprehensive experimental studies have been performed which demonstrated qualitatively and quantitatively the correctness of this theory. At large frequencies (ka $\gtrsim$ 100) where the geometrical fields dominate, studies of the reflection of short acoustic pulses from elastic cylinders, spheres and shells were carried out by Quentin et al [12 - 16]. They verified by a measurement of both the pulse arrival times and their amplitudes, the existence and the predicted properties of transmitted waves, including their multiple reflections and mode conversions. The surface waves on elastic cylinders have been studied extensively by Ripoche et al [17 - 19] at lower values of ka ($\lesssim$ 50), and the predicted dispersion and absorption curves of both Franz-type and elastic-type (internal) surface waves were confirmed experimentally.

2. THEORY OF THE SCATTERING PROCESS.

The total acoustic field in the presence of an infinite elastic cylinder is given in cylindrical coordinates (r,ϕ,z) by

$$p=\sum_{n=0}^{\infty}(2-\delta_{n0})i^n\left\{J_n(kr)+T_n H_n^{(1)}(kr)\right\}\cos n\phi \tag{2.1c}$$

(corresponding to normal incidence of sound ; for oblique incidence, see [20]); for a sphere, it is in spherical coordinates (r,θ,ϕ) :

$$p=\sum_{n=0}^{\infty}(2n+1)i^n\left\{j_n(kr)+T_n h_n^{(1)}(kr)\right\}P_n(\cos\theta). \tag{2.1s}$$

The incident pressure field, given by the first terms in Eqs. (2.1), is here normalized to unity, and T_n is the partial-wave scattering amplitude ("T - function") in the normal-mode or Rayleigh series of Eqs (2.1). It is customary in nuclear physics to rewrite the total amplitude in the form

$$p=\frac{1}{2}\sum_{n=0}^{\infty}(2-\delta_{n0})i^n\left\{H_n^{(2)}(kr)+S_n H_n^{(1)}(kr)\right\}\cos n\phi \tag{2.2c}$$

or

$$p = \frac{1}{2}\sum_{n=0}^{\infty}(2n+1)\,i^{n}\left\{h_n^{(2)}(kr) + S_n h_n^{(1)}(kr)\right\}P_n(\cos\theta), \qquad (2.2s)$$

where

$$S_n = 2T_n + 1, \qquad T_n = \frac{1}{2}(S_n - 1) \qquad (2.3)$$

gives the relation between T_n and the "S - function" S_n. Satisfying the boundary conditions at r = a leads to

$$S_n = S_n^{(s)}\,\frac{F_n - z_n^{(2)}}{F_n - z_n^{(1)}}, \qquad (2.4)$$

with

$$z_n^{(i)} = x\,H_n^{(i)\prime}(x)/H_n^{(i)}(x), \qquad (2.5c)$$

$$z_n^{(i)} = x\,h_n^{(i)\prime}(x)/h_n^{(i)}(x), \qquad (2.5s)$$

i = 1, 2 and x = ka, where

$$S_n^{(s)} = -\,H_n^{(2)}(x)/H_n^{(1)}(x), \qquad (2.6c)$$

$$S_n^{(s)} = -\,h_n^{(2)}(x)/h_n^{(1)}(x) \qquad (2.6s)$$

are the S - functions for a soft scatterer (for rigid scatterers, $H_n^{(i)\prime}$ or $h_n^{(i)\prime}$ appear). The quantities F_n are given in the literature [1]; they are proportional to ρ/ρ_o, the density ratio of the fluid and the scatterer, so that $F \to \infty$ for a soft, and $F \to 0$ for a rigid object.

The Watson transformation [3-5] consists in rewriting the normal-mode sum as a contour integral in the complex n-plane :

$$p = \frac{i}{2}\mathcal{P}\int_{C}\frac{d\nu}{\sin\pi\nu}\,e^{-i\nu\pi/2}\left\{H_\nu^{(2)}(kr) + S_\nu H_\nu^{(1)}(kr)\right\}\cos\nu\phi, \qquad (2.7c)$$

$$p = \frac{1}{2i} \int_C \frac{\lambda\, d\lambda}{\cos \pi\lambda} \left\{ h^{(2)}_{\lambda-\frac{1}{2}}(kr) + S_{\lambda-\frac{1}{2}}\, h^{(1)}_{\lambda-\frac{1}{2}}(kr) \right\} P_{\lambda-\frac{1}{2}}(-\cos\theta), \tag{2.7s}$$

where C tightly surrounds the positive real axis in the complex ν or λ plane, passing through $\nu = 0$ (P = principal value) but to the right of $\lambda = 0$. The Imai transformation

$$\cos \nu\phi = e^{i\nu\pi} \cos \nu(\phi - \pi) - i e^{i\nu(\pi-\phi)} \sin \pi\nu, \tag{2.8c}$$

$$P_{\lambda-\frac{1}{2}}(-\cos\theta) = e^{-i\pi(\lambda-\frac{1}{2})} P_{\lambda-\frac{1}{2}}(\cos\theta) - 2i \cos\pi\lambda\; Q^{(-)}_{\lambda-\frac{1}{2}}(\cos\theta), \tag{2.8s}$$

where

$$Q_\mu^{(\pm)}(\cos\theta) = \frac{1}{2}\left\{ P_\mu(\cos\theta) \mp \frac{2i}{\pi} Q_\mu(\cos\theta) \right\} \tag{2.9}$$

defines $Q_\mu^{(\pm)}$ in terms of the Legendre function of the second kind, Q_μ, splits Eqs. (2.7) into two portions (note that the terms with $H_\nu^{(2)}$ or $h^{(2)}_{\lambda-1/2}$ integrate to zero). The first one, containing $\cos \nu(\phi - \pi)$ or $P_{\lambda-1/2}(\cos\theta)$, can be evaluated at the poles of the S-function S_ν in the ν - plane("Regge poles"), given by the zeros of $F_\nu - z_\nu^{(1)}$ and denoted by $\nu = \nu_\ell$ (ℓ = 1,2,3...labeling their multiplicity) :

$$p_{cw} = \sum_{\ell=1}^{\infty} \frac{\pi}{\sin \pi\nu_\ell} e^{i\pi\nu_\ell/2} S^{(R)}_{\nu_\ell} H^{(1)}_{\nu_\ell}(kr) \cos \nu_\ell(\phi - \pi), \tag{2.10c}$$

$$p_{cw} = \sum_{\ell=1}^{\infty} \frac{\pi(\nu_\ell + \frac{1}{2})}{\sin \pi\nu_\ell} e^{-i\pi\nu_\ell} S^{(R)}_{\nu_\ell} h^{(1)}_{\nu_\ell}(kr) P_{\nu_\ell}(\cos\theta), \tag{2.10s}$$

where $S_{\nu_\ell}^{(R)}$ is the residue of S_ν at $\nu = \nu_\ell$. The exponential form of $\cos \nu_\ell (\phi - \pi)$, and the asymptotic form

$$P_{\nu_\ell}(\cos\theta) \sim \left\{ \frac{2}{\pi(\nu_\ell + \frac{1}{2})\sin\theta} \right\}^{1/2} \sum_{\varepsilon=\pm 1} e^{i\varepsilon(\nu_\ell + \frac{1}{2})\theta - i\varepsilon\frac{\pi}{4}} \tag{2.11}$$

demonstrate that Eqs. (2.10) represent circumferential waves p_{cw} with propagation constant ν_ℓ for the cylinder, $\nu_\ell + 1/2$ for the sphere. From this, one finds the phase velocities

$$c_\ell^c = \frac{x}{\mathrm{Re}\,\nu_\ell} c, \qquad c_\ell^s = \frac{x}{\mathrm{Re}\,\nu_\ell + \frac{1}{2}} c, \tag{2.12a}$$

and the attenuation angles (amplitude $\exp -\phi/\phi_\ell$):

$$\phi_\ell = \theta_\ell = \frac{1}{\mathrm{Im}\,\nu_\ell} \tag{2.12b}$$

of the surface waves, both the external and the internal ones.

The second part of Eqs. (2.7) is

$$p_{gw} = \frac{1}{2}\int_C e^{i\nu(\frac{\pi}{2} - \phi)} S_\nu H_\nu^{(1)}(kr)\, d\nu, \tag{2.13c}$$

$$p_{gw} = -\int_C \lambda S_{\lambda-\frac{1}{2}} h_{\lambda-\frac{1}{2}}^{(1)}(kr)\, Q_{\lambda-\frac{1}{2}}^{(-)}(\cos\theta)\, d\lambda. \tag{2.13s}$$

These integrals can be evaluated at the saddle points of the integrand [3,4,8,9] and are then seen to represent geometrical waves p_{gw} , either reflected from the apex of the scatterer or undergoing internal transmissions without and with multiple internal reflections, including mode conversions for an elastic scatterer. This is obtained from an expansion of S_ν in Eqs. (2.13) into a Debye series, as was done for an elastic cylinder by Brill and Überall [9], but which for the simpler case of fluid scatterers [8,10] becomes, e.g. for cylinders :

$$S_\nu = S_\nu^{(s)}\left\{R_{12} - \frac{H_\nu^{(1)}(\beta x)}{H_\nu^{(2)}(\beta x)} T_{12} T_{21} \sum_{k=1}^{\infty} \left(\frac{H_\nu^{(1)}(\beta x)}{H_\nu^{(2)}(\beta x)} R_{21} \right)^{k-1} \right\}, \tag{2.14a}$$

where $\beta = c/c_o$, the sound velocity ratio of fluid and scatterer. This contains an external reflection coefficient

$$R_{12} = \left\{ \frac{H_\nu^{(2)\prime}(x)}{H_\nu^{(2)}(x)} - N \frac{H_\nu^{(2)\prime}(\beta x)}{H_\nu^{(2)}(\beta x)} \right\} U^{-1} \tag{2.14b}$$

and an internal one,

$$R_{21} = -\left\{ \frac{H_\nu^{(1)\prime}(x)}{H_\nu^{(1)}(x)} - N \frac{H_\nu^{(1)\prime}(\beta x)}{H_\nu^{(1)}(\beta x)} \right\} U^{-1} \tag{2.14c}$$

where

$$U = \frac{H_\nu^{(1)\prime}(x)}{H_\nu^{(1)}(x)} - N \frac{H_\nu^{(2)\prime}(\beta x)}{H_\nu^{(2)}(\beta x)} , \tag{2.14d}$$

as well as the transmission coefficients

$$T_{12} = 1 - R_{12} , \quad T_{21} = 1 + R_{21} \tag{2.14e}$$

into and out of the object, respectively ; also, $N = \beta\rho/\rho_o$. For the sphere, one replaces $H_\nu^{(i)} \to h_\nu^{(i)}$ in Eqs. (2.14). The individual terms in Eqs. (2.14), evaluated at their saddle points in Eqs. (2.13), furnish the geometrical rays corresponding to no (R_{12}) or k internal traversals.

3 EXPERIMENTAL RESULTS.

The surface waves form resonating standing waves around the scatterer for Re $\nu_\ell = n$, where n (n+1/2) of their wavelengths span the cylinder (sphere ;

here, a $\lambda/4$ phase jump takes place at each of their two convergence points [21]). By observing these resonances on cylinders, Ripoche et al [17-19] have verified the surface waves, and determined their phase velocities c_ℓ^c and attenuations ϕ_ℓ, Eqs. (2.12). Figure 3.1 shows their results (crosses) for the attenuation of Whispering Gallery waves (internal, $\ell = 2,3$) as compared to theory [7] (curves).

For the transmitted waves in cylinders and spheres, experiments [12,13] give excellent agreement with the theoretical arrival times and amplitudes [9]. Figure 3.2 shows measured and calculated arrival times of short acoustic pulses backscattered from a lucite cylinder (the inserts show the type of traversals).

4. CONCLUSION

The theory of sound scattering from elastic cylinders and spheres, as based on the Watson transformation, gives an accurate account of the physics of the diffraction phenomenon. Its various aspects, i.e. the existence and properties of (resonating) external and internal circumferential waves, and of geometrically reflected and internally transmitted rays, have all been verified experimentally. Analogous studies with scatterers of more general shapes are now called for.

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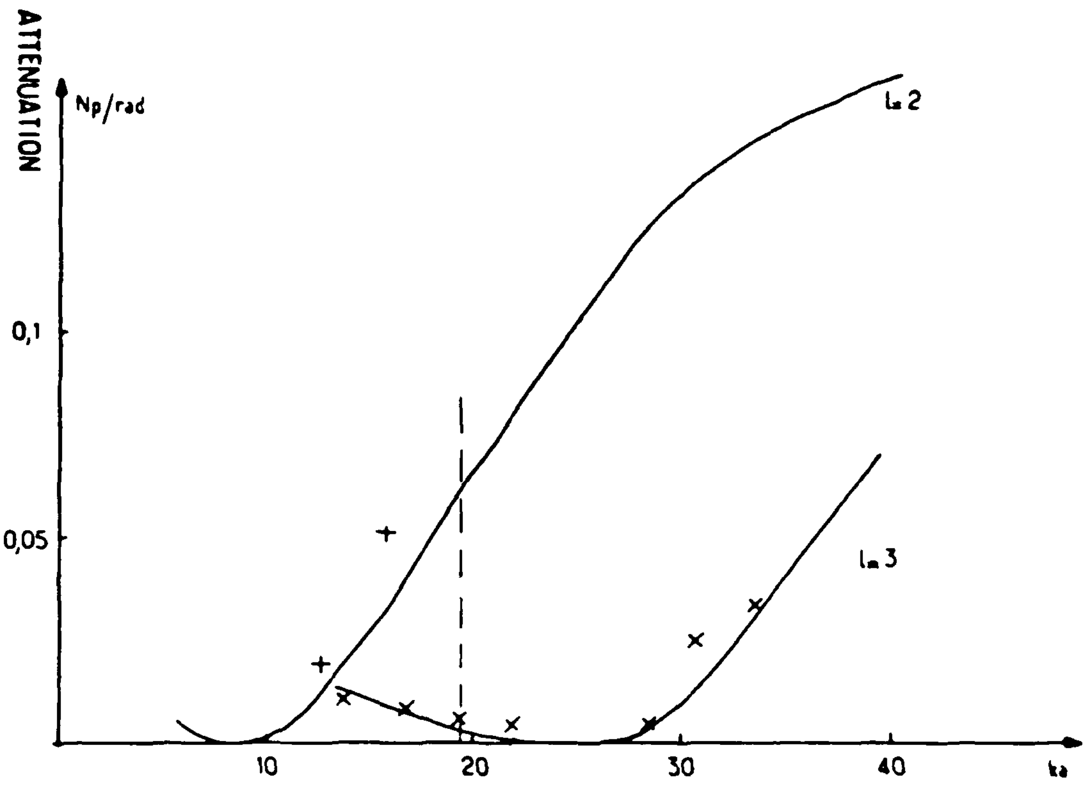
FIGURE CAPTIONS.

Fig. 3.1. Attenuation of the whispering Gallery waves ℓ = 2, 3. Solid curves : theory [7] ; crosses : experiment [19].

Fig. 3.2. Experimental [12] and theoretical [9] arrival times of short sound pulses traversing a lucite cylinder (Solid rays : dilatational ; broken rays : shear).

Fig. 3.1


ATTENUATION
Np/rad
l=2
0,1
0,05
l=3
10
20
30
40
ka

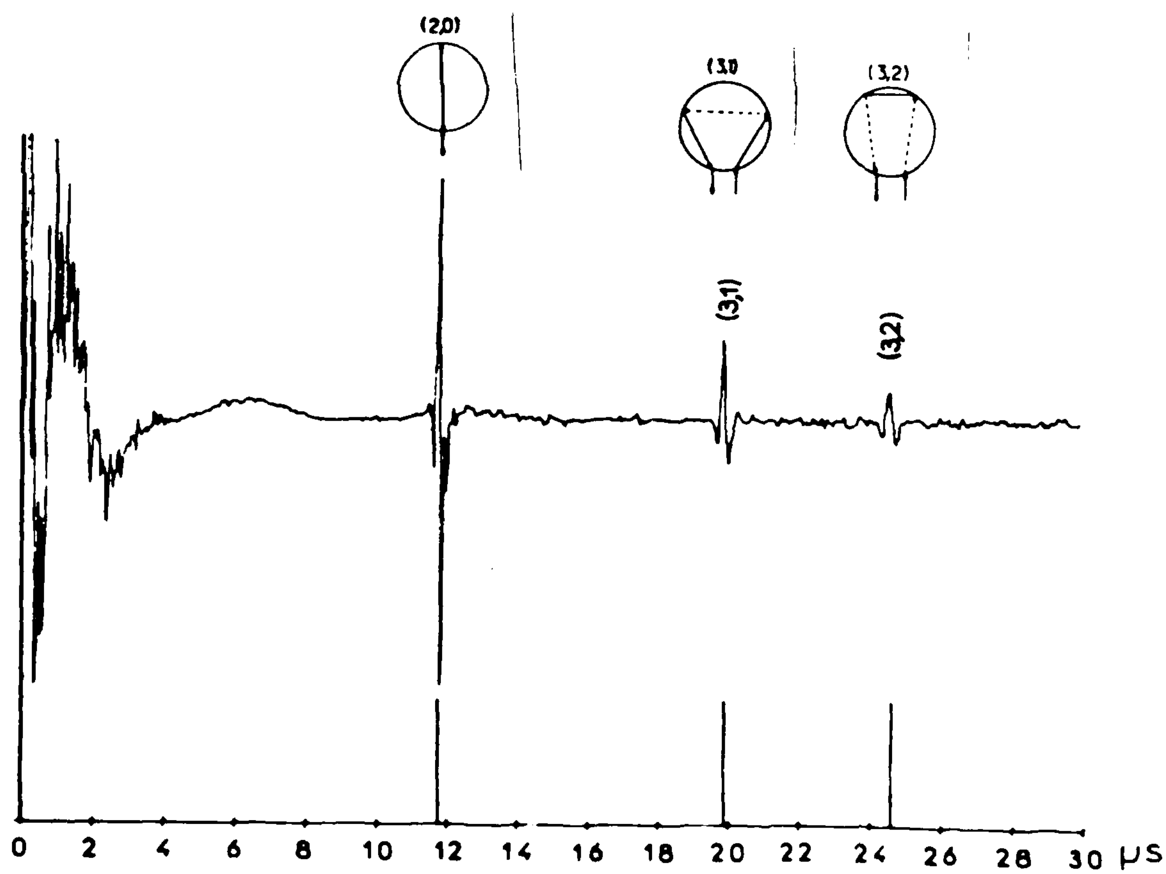

(2,0)
(3,1)
(3,2)
(3,1)
(3,2)
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30 μs

Fig. 3.2

DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

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HELICAL SURFACE WAVES ON CYLINDERS AND CYLINDRICAL CAVITIES.

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SUMMARY : The eigenfrequencies of a cylindrical obstacle (of finite or infinite length) can be interpreted as the resonances due to phase matching of circumnavigating helical surface waves. For the case of a cylinder of finite length, the pitch angle of the helix can assume a discrete set of values only. Resonant eigenvibrations can be excited by waves incident in an oblique fashion, which generates the helical waves. A refraction effect is found to take place between the incident and the helical-wave directions. We obtain pole diagrams of the scattering amplitude in the complex-frequency plane, by using the T-matrix approximation for finite cylinders. In addition, pole diagrams for spheroidal scatterers are obtained by the use of the T-matrix and of spheroidal wave functions. While the poles of symmetric scatterers (spheres or infinite cylinders) are degenerate in the azimuthal quantum number m, the degeneracy for the poles of finite cylinders and of spheroids is lifted. This m-splitting is explained by the phase matching of helical waves with various allowed pitch angles. Dispersion curves for the phase and group velocities and attenuations of the helical waves are obtained.

RESUME : Les fréquences propres d'un obstacle cylindrique (de longueur finie ou infinie) peuvent être interprétées comme dues à l'accord entre les phases d'ondes se propageant sur la surface d'une façon hélicoïdale. Dans le cas d'un cylindre de longueur finie, l'angle de pas de l'hélice ne peut prendre qu'une série de valeurs discrètes. Des vibrations propres résonnantes peuvent être excitées par des ondes incidentes de direction oblique, ce qui produit les ondes hélicoïdales. Un effet de réfraction est trouvé entre les directions de l'onde incidente et de l'onde hélicoïdale. On obtient des diagrammes de pôles dans l'amplitude de diffusion dans le plan complexe de la fréquence, par un calcul utilisant l'approximation de la matrice T pour des cylindres finis.

En plus, on obtient des diagrammes de pôles pour des obstacles sphéroïdaux en utilisant la matrice T, ou des fonctions d'ondes sphéroïdales. Tandis que les pôles d'obstacles symètriques (sphères, ou cylindres infinis) dégénérent vis-à-vis du nombre quantique azimuthal m, cela n'est plus le cas pour les pôles de cylindres finis et de sphéroïdes. La séparation résultante entre les valeurs de m s'explique alors par l'accord de phases des ondes hélicoïdales possédant différents angles d'inclinaison permis. On obtient des courbes de dispersion pour les vitesses de phase et de groupe des ondes hélicoïdales.

1. INTRODUCTION.

The most general case of the scattering of waves obliquely incident on an infinite cylinder has first been considered by White [1] ; he deals with a plane wave in an elastic medium incident on an elastic cylindrical inclusion. This theory has recently been completed by Delsanto et al [2,3] ; it is based on the classical normal-mode (or partial-wave) Rayleigh series expansion. Special cases have been investigated in greater detail : acoustic waves incident on an elastic cylinder [4] or shell [5], or on a fluid cylinder [6,7], and elastic waves incident on a fluid-filled cylindrical cavity [8] (in a similar fashion, elastic waves on a spherical cavity have also been studied [9]). In these latter investigations, the complex eigenfrequencies of the mentioned obstacles have been explicitly calculated, as well as the excitation of the corresponding eigenvibrations. While analogous calculations cannot be carried out for the case of cylinders of finite length, due to the non-separability of this problem, results for the eigenfrequencies can nevertheless be obtained here by the use of special methods. If only the interior vibrations of a fluid cylinder in vacuo are considered, the problem is still exactly soluble, and the corresponding eigenvibrations have, for the first time, been interpreted as being due to the phase matching of helical surface waves [10] (of internal type, in this case). For the non-separable exterior problem of finite-length impenetrable cylinders, the T-matrix method of Waterman [11] has been employed, modified so as to furnish complex eigenfrequencies [12]. A very comprehensive study of this problem has recently been carried out, on the basis of this method, which is contained in a paper [13] that also discusses the acoustic eigenfrequencies of impenetrable spheroids obtained by the use of spheroidal functions. In this study, the phase matching of external helical surface waves has been invoked as an explanation for the finite-cylinder eigenfrequencies, and for their splitting

into components corresponding to different values of the azimuthal quantum number m (the latter being a measure for the pitch angles of the helical waves, which due to the finite cylinder length form a discrete set). As to the excitation of helical surface waves by incident sound, if was found [6,13] that this takes place in a refractive way, the helical pitch angle being different from the incident angle. Experiments on the scattering of obliquely incident sound by elastic cylinders have recently been initiated [14].

2. INTERNAL HELICAL WAVES.

A fluid cylinder in vacuo, of radius a and length L admits an internal acoustic field (wave vector $\vec{k}$, with $k = \omega/c$) :

$$p(\vec{r}) = J_m(\varkappa r)\, e^{\pm i k_\phi a \phi}\, e^{\pm i k_z z} \tag{2.1}$$

where

$$\varkappa^2 = k^2 - k_z^2 \; ; \tag{2.2a}$$

the single - valuedness condition under $\phi \rightarrow \phi + 2\pi$ gives

$$k_\phi = m/a, \qquad m = 1, 2, \cdots , \tag{2.2b}$$

and the boundary condition at the end faces leads to

$$k_z = j\pi/L, \qquad j = 1, 2, \ldots . \tag{2.2c}$$

The Dirichlet boundary condition at $r = a$ gives

$$\varkappa = x_{mn}/a, \qquad n = 1, 2, \ldots \tag{2.2d}$$

where x_{mn} is the nth zero of $J_m(x)$. [The Neumann condition for a fluid cylinder in a rigid enclosure would lead to the zero of $J'_m(x)$]. Inserted in Eq. (2.2a), this gives the eigenvalues k_{nmj} of k.

We now introduce a tangential wave vector $\vec{k}_s$ of the surface field, with

$$k_s^2 = k_\phi^2 + k_z^2 , \tag{2.3}$$

which describes the propagation of helical surface waves. The conditions (2.2b) and (2.2c) then represent the phase matching of such waves after circumnavigating the cylinder, and/or getting reflected from the end faces. Equations (2.2b), (2.2c) inserted in Eq. (2.3) furnish eigenvalues $(k_s)_{mj}$. The helical-wave phase velocities $c_s = \omega/k_s$ can be obtained from $c_s/c = k/k_s$ at the discrete points (resonance frequencies) where phase matching is satisfied :

$$(c_s/c)_{nmj} = \{[x_{mn}^2 + (j\pi)^2(a/L)^2]/[m^2 + (j\pi)^2(a/L)^2]\}^{1/2}. \tag{2.4}$$

Note that these correspond to helical waves of pitch angle α with the z - axis, $\tan\alpha = k_\phi/k_z$, which due to the finite length of the cylinder assumes the discrete values

$$\tan\alpha_{mj} = (m/j\pi)(L/a). \tag{2.5}$$

A given helical wave thus corresponds to a fixed ratio m/j. For an infinite cylinder ($L \to \infty$, $j \to \infty$) α is continuous.

Figure 2.1 shows how the discrete points of Eq. (2.4), corresponding to the eigenfrequencies of the cylinder, when connected according to Eq. (2.5) furnish the dispersion curves of the helical surface waves.

3. REFRACTION EFFECT.

For a cylinder in a medium, helical waves can be generated by an incident plane acoustic wave. If the latter arrives at an angle γ with the z-axis so that $k_y = k \sin\gamma$, $k_z = k \cos\gamma$, the total field for an infinite cylinder, given by Eq. (2.2c) of Ref. [15], gets modified to

$$p = \frac{1}{2} e^{ik_z z} \sum_{n=0}^{\infty} (2-\delta_{n0}) i^n \{H_n^{(2)}(k_y r) + S_n H_n^{(1)}(k_y r)\} \cos n\phi, \tag{3.1}$$

leading after application of the Watson transformation to the creeping-wave sum analogous to Eq. (2.10c) of [15] :

$$p_{cw} = \pi e^{ik_z z} \sum_{\ell=1}^{\infty} \frac{e^{i\pi\nu_\ell/2}}{\sin\pi\nu_\ell} S_{\nu_\ell}^{(R)} H_{\nu_\ell}^{(1)}(k_y r) \cos\nu_\ell(\phi-\pi). \tag{3.2}$$

The phase factor $\Phi_\ell = k_z z + \phi \operatorname{Re} \nu_\ell$ shows that these surface waves are helical, with wave fronts $a\phi = -(ak_z/\operatorname{Re} \nu_\ell) z$ whose normals make an angle $\phi_\ell = \tan^{-1} (\operatorname{Re} \nu_\ell/ak_z)$ with the z- axis. This defines the law of refraction

$$\tan\phi_\ell = g_\ell \tan\gamma, \qquad g_\ell = \operatorname{Re} \nu_\ell / k_y a \tag{3.3}$$

between incident direction γ and helical-wave direction ϕ_ℓ. The phase velocity $v_\ell^{ph} = ck/k_\ell$ of the helical waves is

$$v_\ell^{ph} = c/\{(\operatorname{Re} \nu_\ell/ka)^2 + \cos^2\gamma\}^{1/2}; \tag{3.4}$$

for the case of external waves on rigid or soft cylinders where the asymptotic expansion of Franz [16] for $\nu_\ell(k_y a)$ can be used, one has

$$v_\ell^{ph} \cong c/\left\{1 + \frac{q_\ell}{2.6^{1/3}}\left(\frac{\sin^2\gamma}{ka}\right)^{2/3} + \cdots\right\} \tag{3.5}$$

where $q_1^r = 1.469354$, $q_1^s = 3.372134$. Including higher terms, Fig. 3.1 shows v_ℓ^{ph} of helical waves at $\gamma=0°$ and $45°$, and the refraction angle ϕ_ℓ for $\gamma = 45°$, for a soft cylinder.

4. COMPLEX EIGENFREQUENCIES.

The eigenfrequencies k_{nmj} for a cylinder in vacuo, Section 3, are real since no radiation loss can occur. We present figures showing examples of complex eigenfrequencies that correspond to the resonances, due to phase matching, of external circumferential waves on elongated objects.

Figure 4.1 shows the eigenfrequencies in the complex kb plane of an acoustically rigid prolate spheroid, for various axis ratios b/a as indicated (a being the semiminor axis). These were obtained [13] by satisfying the

boundary condition with spheroidal wave functions. One notices that the sphere values (crossed squares), degenerate in m, split into branches non-degenerate in m for spheroids. Figure 4.2 shows the same for the magnetic eigenvibrations of a conducting cylinder of length L (the electromagnetic analogue of an acoustically rigid cylinder) obtained with the T-matrix method [11-13]. A phase-matching model for surface waves has been developed here [13] in order to show that the m-splitting of the eigenfrequencies corresponds to helices of different pitch angles.

5. CAVITIES.

Analogous results were obtained for infinite cylindrical cavities, using our general theory [2,3]. Figure 5.1 shows the eigenfrequencies in the complex $k_y a$ plane for an empty cavity in aluminum, for both compressional (p) and shear (s) type surface waves (here, k is the dilatational propagation constant).

6. CONCLUSION.

The complex eigenfrequencies of finite-length cylinders show a splitting according to the azimuthal quantum number m. They can be interpreted as the resonances, due to phase matching, of helical surface waves of different pitch angles. For infinite cylinders, a continuum of pitch angles occurs. The helical surface waves can be excited by incident acoustic waves, and refraction takes place between the incident and the surface wave directions. Recent experiments [14] are now investigating these problems. Theoretically, there have also been geometrical investigations of helical waves on cylinders [17].

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FIGURE CAPTIONS.

Fig. 2.1. Dispersion curves of internal helical surface waves on a finite fluid cylinder in vacuo, of dimension 1:1 ($L = 2a$).

Fig. 3.1. Dispersion curves of external helical surface waves on an infinite soft cylinder for $\gamma = 90°$ (dashed) and 45° (solid), and refraction angle ϕ_ℓ at $\gamma = 45°$ (right scale), plotted vs. $k_y a$.

Fig. 4.1. Complex eigenfrequencies for a rigid spheroid with semi-major axis b.

Fig. 4.2. Complex magnetic eigenfrequencies of a conducting cylinder of length L.

Fig. 5.1. Eigenfrequencies in the complex $k_y a$ plane for an empty cylindrical cavity in aluminum.

Fig. 2.1

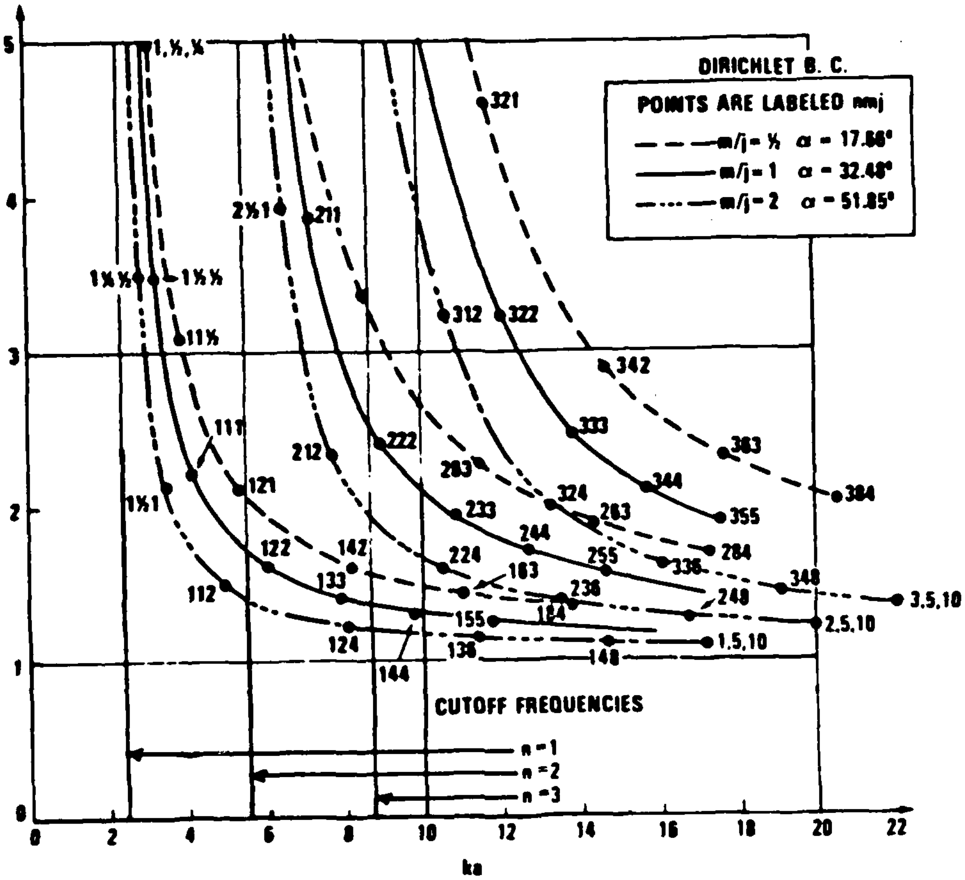

DIRICHLET B. C.
POINTS ARE LABELED nmj
m/j = ½ α = 17.66°
m/j = 1 α = 32.48°
m/j = 2 α = 51.85°
CUTOFF FREQUENCIES
n = 1
n = 2
n = 3
ka

Fig 3.1

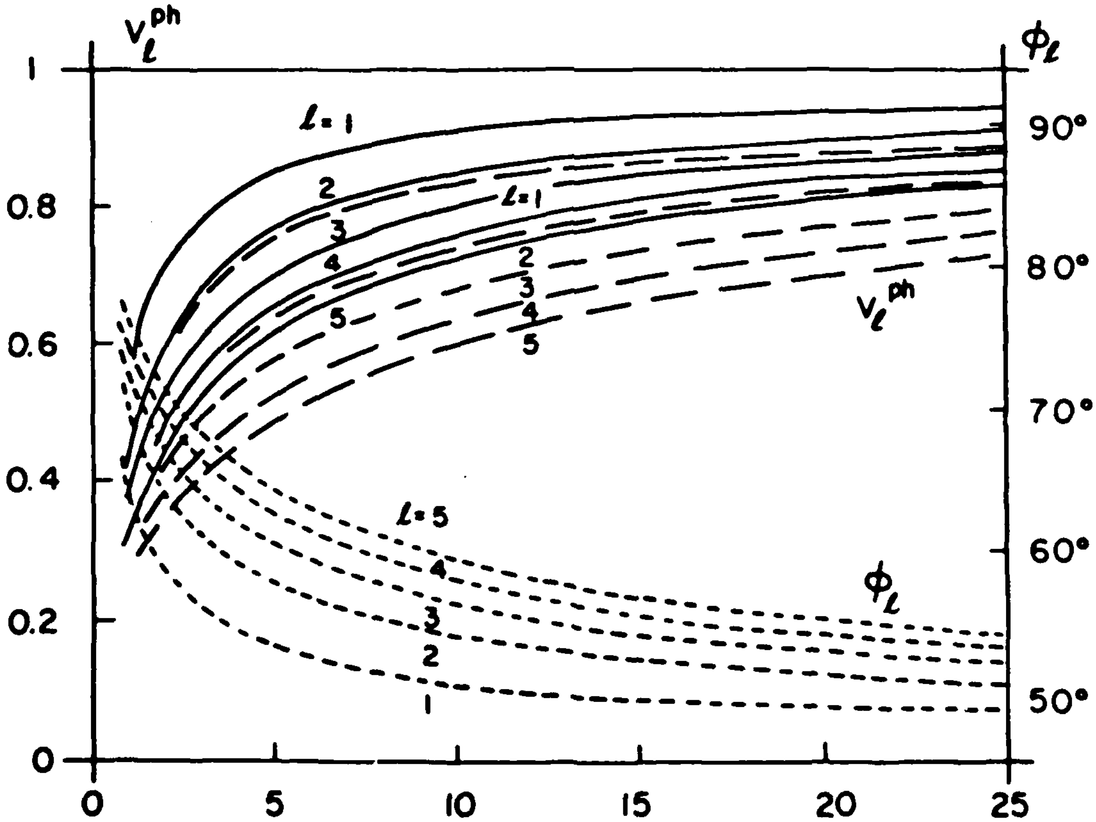

V_ℓ^{ph}
ϕ_ℓ
90°
80°
70°
60°
50°

Fig. 4.1

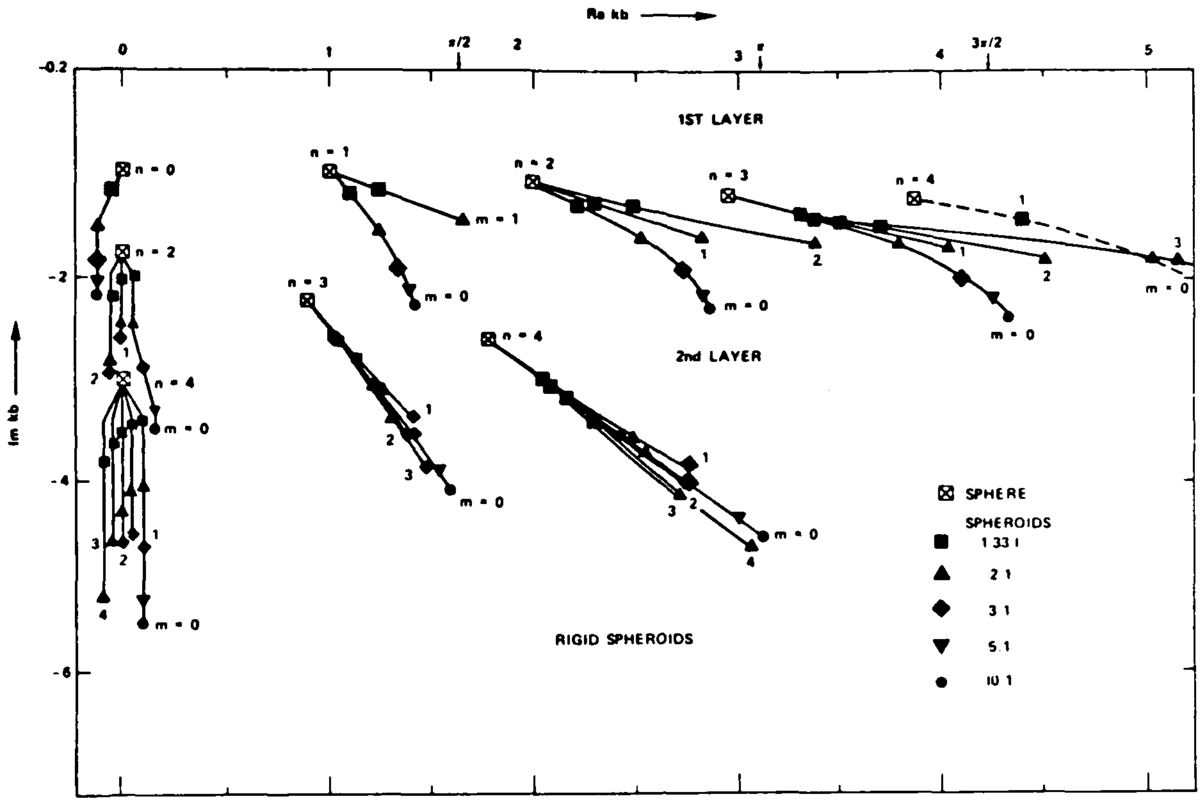

Re kb
Im kb
1ST LAYER
2nd LAYER
RIGID SPHEROIDS
SPHERE
SPHEROIDS
1.33:1
2:1
3:1
5:1
10:1

Fig. 4.2

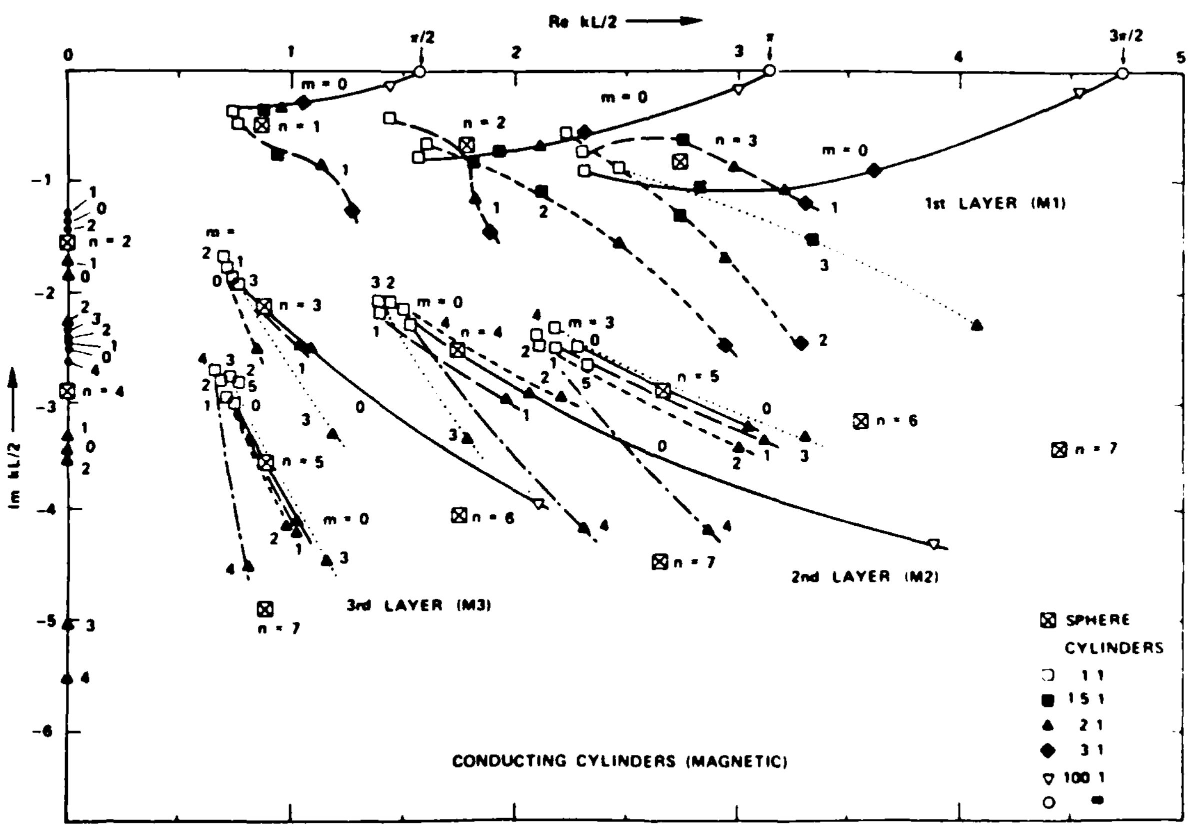
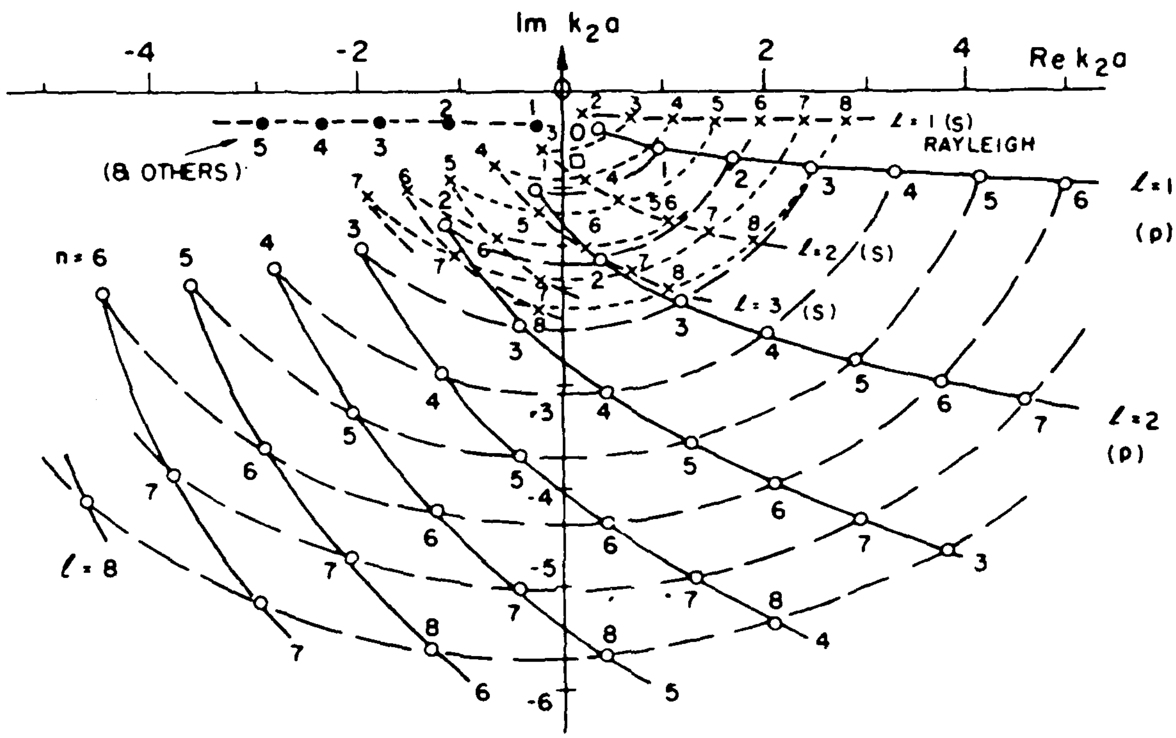

Re kL/2
Im kL/2
1st LAYER (M1)
2nd LAYER (M2)
3rd LAYER (M3)
CONDUCTING CYLINDERS (MAGNETIC)
SPHERE
CYLINDERS
1:1
1.5:1
2:1
3:1
100:1
∞

Fig. 5.1


Im k_2a
Re k_2a
-4
-2
2
4
-3
-4
-5
-6
$\ell = 1$ (S)
RAYLEIGH
(8 OTHERS)
$\ell = 1$
(p)
$\ell = 2$ (S)
$\ell = 3$ (S)
$\ell = 2$
(p)
n = 6
$\ell = 8$

DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

Université Paris VII

4 - 7 Décembre 1984

RESONANCE SCATTERING THEORY : SPHERICAL AND CYLINDRICAL CAVITIES AND INCLUSIONS

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SUMMARY : We consider the scattering of elastic dilatational and shear waves from cylindrical and spherical cavities and inclusions in an elastic medium. The normal mode series of the scattering amplitude is reformulated in terms of the S- function, and the poles of the S- function in the complex frequency plane are identified. The amplitude is rewritten as a "background term" including specular reflections and external surface waves, plus a series of (internal) resonance terms. This formulation is termed the "Resonance Scattering Theory" (RST). The connection between the resonances and the surface waves is established via expressing the complex-frequency poles of the scattering amplitude by the Regge poles in the complex-mode number plane, and the frequency resonances in successive modes are recognized as the Regge recurrences of surface-wave resonances. This permits us to obtain the dispersion curves of phase and group velocities of the (internal) surface waves from the eigenfrequencies of the cavity. We also mention experiments on ultrasonic scattering from cavities and inclusions.

RESUME : Nous étudions la diffusion d'ondes élastiques de type dilatation ou cisaillement par des cavités et inclusions cylindriques ou sphériques. La série des modes normaux donnant l'amplitude de diffusion est recomposée en utilisant la fonction S, et les pôles de la fonction S dans le plan complexe des fréquences sont identifiés. L'amplitude est réécrite sous la forme d'un "terme de fond", comprenant réflexions spéculaires et ondes de surface externes, plus une série de termes de résonances (internes). Ce formalisme est appelé " Théorie de diffusion résonnante" ["Resonance Scattering Theory" (RST)]. Le rapport entre les résonances et les ondes de surface est établi

en exprimant les pôles en fréquence complexe de l'amplitude de diffusion par les pôles de Regge dans le plan complexe du numéro de mode, et les résonances en fréquence dans les modes successifs sont reconnues comme les récurrences des résonances d'ondes de surface. Cela nous permet d'obtenir les courbes de dispersion pour la vitesse de phase et de groupe des ondes (internes) de surface, en utilisant les fréquences propres de la cavité. Nous mentionnons également des expériences sur la diffusion ultrasonore par des cavités et inclusions.

1. INTRODUCTION

The Resonance Scattering Theory (RST) has first been formulated ([1] ; see also [2,3]) for the case of sound scattering from a solid cylinder or sphere in a fluid. The resonance frequencies, which may e.g. be represented by a "pole diagram" in the complex frequency plane [4,5], were shown to originate from the phase matching of surface waves [2,3,6]. When the frequency poles are transferred to the complex mode number plane [6], where they constitute the "Regge poles", they determine by their real and imaginary parts the propagation constant and attenuation of the surface waves, respectively ; hence, the dispersion and attenuation curves of the surface waves can be obtained from the resonance frequencies without having to go through the (complex) Watson transformation.

A similar program may be carried out for ultrasonic wave scattering from cavities of cylindrical [7-9] or spherical [10-13] shape, or from solid inclusions [14,15]. This topic is chosen here to present the resonance theory, since surface waves and resonances in acoustic scattering from elastic objects are treated elsewhere in this issue [16,17]. While experiments on resonant acoustic scattering from solids are quite numerous by now (see, e.g., [18,19] and references quoted therein), ultrasonic experiments on scattering from cavities or inclusions as done so far [20-23] have not been specifically designed to detect resonances, so that resonance effects appear in their results only indirectly.

2. RESONANCE THEORY.

We consider an infinite-cylindrical cavity in a solid (density ρ, wave speeds c_p, c_s) filled with a fluid (density ρ_f, sound speed c_f). The displacement field $\vec{u}$ in the solid is represented by a scalar (Φ) and a vector ($\vec{\Psi}$) potential :

$$\vec{u} = \vec{\nabla}\Phi + \vec{\nabla} \times \vec{\Psi} \tag{2.1}$$

(with $\vec{\nabla} \cdot \vec{\Psi} = 0$), and a normally incident p-wave is given by

$$\Phi_{inc} = e^{i(k_p x - \omega t)} = \sum_{n=0}^{\infty} (2-\delta_{n0}) i^n J_n(k_p r) \cos n\phi, \tag{2.2}$$

where $k_p = \omega/c_p$ (suppressing exp$-i\omega t$). Scattered elastic waves are of both p-type,

$$\Phi_{sc} = \frac{1}{2} \sum_{n=0}^{\infty} (2-\delta_{n0}) i^n (S^{pp} - 1) H_n^{(1)}(k_p r) \cos n\phi, \tag{2.3}$$

or s- type ("mode conversion"),

$$\Psi_{sc} = \frac{1}{2} \sum_{n=0}^{\infty} (2-\delta_{n0}) i^n S_n^{ps} H_n^{(1)}(k_s r) \sin n\phi \tag{2.4}$$

where $k_s = \omega/c_s$. A standing wave exists in the fluid filler :

$$\Phi_f = \frac{1}{2} \sum_{n=0}^{\infty} (2-\delta_{n0}) i^n C_n^p J_n(k_f r) \cos n\phi, \tag{2.5}$$

where $k_f = \omega/c_f$. Similar equations hold [8] for s-wave incidence (assumed SV, so that only $\Psi \equiv \Psi_z \neq 0$), with coefficients S_n^{sp}, S_n^{ss} and C_n^s. The sets of S_n^{ik} form the 2x2 S-matrix.

Satisfying the boundary conditions (continuity of normal stresses and displacements, zero tangential stress) at the cylinder radius r = a leads to 3x3 linear systems

$$D_n^p \vec{S}_n^p = \vec{E}_n^p, \qquad D_n^s \vec{S}_n^s = \vec{E}_n^s \tag{2.6a}$$

where $D_n^{p,s}$ are 3x3 matrices and $\vec{E}_n^{p,s}$ vectors, all with known elements [8] which consist of cylinder functions of order n and arguments $\alpha \equiv k_p a$, $\beta \equiv k_s a$ and $\gamma \equiv k_f a$, and

$$\vec{S}_n^{\,p} = \begin{pmatrix} S_n^{pp} \\ S_n^{ps} \\ C_n^{p} \end{pmatrix}, \qquad \vec{S}_n^{\,s} = \begin{pmatrix} -S_n^{sp} \\ S_n^{ss} \\ C_n^{s} \end{pmatrix}. \tag{2.6b}$$

Using Cramer's rule, the elements S_n^{ik} are thus fractions with $D_n(\omega) \equiv \det(D_n^{p}) \equiv -\det(D_n^{s})$ in the denominator. Resonances occur when $D_n(\omega) = 0$; the complex solutions ω_{nj} ($j = 1,2,\ldots$) represent resonance frequencies common to all four amplitudes S_n^{ik}. Alternately, at a given ω (real), $D_n(\omega) = 0$ is an equation for n with complex solutions $n = \nu_j(\omega)$. While ω_{nj} are complex-frequency poles ("SEM poles", see [17]) of the scattering amplitude, $\nu_j(\omega)$ are its complex mode number ("Regge") poles [16] ; both are related through $D_\nu(\omega) = 0$.

More explicitly, one may write for S_n^{ik}, e.g.,

$$S_n^{pp} = S_n^{(0)pp}\,\frac{Z_n^{(2)} - K_n}{Z_n^{(1)} - K_n} \tag{2.7a}$$

where

$$K_n = \frac{\rho}{\rho_f}\,\frac{\gamma J_n'(\gamma)}{J_n(\gamma)}\ , \tag{2.7b}$$

with $Z_n^{(i)}$ expressed by the elements of D_n^{p}, and $S_n^{(o)pp} \equiv \exp 2i\delta_n$ the value of S_n^{pp} for an empty cavity ($\rho_f = 0$). Expanding $K_n(\alpha)$ about the resonance value $\alpha_{nj} \equiv \omega_{nj} a/c_p$ leads to

$$S_n^{pp} - 1 \cong e^{2i\delta_n}\left\{\sum_{j=1}^{\infty} \frac{M_{nj}}{\alpha - \alpha_{nj} + \frac{i}{2}\Gamma_{nj}} + 2i\,e^{-i\delta_n}\sin\delta_n\right\} \tag{2.8}$$

where

$$M_{nj} = (Z_n^{(1)} - Z_n^{(2)})/K'_n(\alpha_{nj}), \tag{2.9a}$$

$$\Gamma_{nj} = -2\,\mathrm{Im}\; Z_n^{(1)}/K'_n(\alpha_{nj}). \tag{2.9b}$$

This is the Breit-Wigner form of the scattering amplitude, as developed in nuclear scattering [24]. It consists of a sum of resonant terms (with M_{nj}: internal resonances), and a smooth background term corresponding to the scattering from an empty cavity, second term in Eq. (2.8). However, this background still contains resonances corresponding to external (Franz) waves, as well as geometrical (specurlarly reflected, and transmitted) waves [16].

The quantities $\nu_j(\omega)$ are the propagation constants of surface waves, as in the Watson-transform method [16]. This can be seen, e.g., by expressing the resonance denominator in α in Eq. (2.8) by one linear in n (assumed a continuous variable) [6, 25] ; this furnishes the Watson results without having to go through the complex arithmetic. The surface wave parameters are thus

$$c_j^{ph} = a\omega/\mathrm{Re}\,\nu_j(\omega) \tag{2.10a}$$

(phase velocity),

$$c_j^{gp}(\omega) = a/\mathrm{Re}\,(d\nu_j/d\omega) \tag{2.10b}$$

(group velocity), and(amplitude exp $-\phi/\phi_\ell$) :

$$\phi_\ell = 1/\mathrm{Im}\,\nu_\ell \tag{2.10c}$$

(attenuation). Their wavelength (in the fluid) is

$$\lambda_j = \frac{2\pi a}{\mathrm{Re}\,\nu_j}\quad , \tag{2.10d}$$

showing that a resonance ($\mathrm{Re}\,\nu_j \to n$) occurs when the circumference of the cavity equals an integral number of wavelengths of the surface wave ("phase matching").

For the internal surface waves which have small imaginary parts, one has at resonance ($\omega \cong \mathrm{Re}\,\omega_{nj}$) approximately :

$$c_j^{ph}(\mathrm{Re}\,\omega_{nj}) \cong a\,\mathrm{Re}\,\omega_{nj}/n, \tag{2.11a}$$

$$c_j^{gp}(\mathrm{Re}\,\omega_{nj}) \cong a\,d\mathrm{Re}\,\omega_{nj}/dn \tag{2.11b}$$

(to be understood as a difference quotient), so that the dispersion curves of surface waves at the resonance points can be directly read off trom the latter.

A similar theory was developed for the case of scattering from spherical [14] and cylindrical [15] inclusions.

3. NUMERICAL RESULTS.

The (internal) frequency resonances appear in a plot, e.g. of $|S_n^{pp} - 1|$, vs. $k_p a$, as shown in Fig. 3.1(a) for a water-filled cylindrical cavity in aluminum. Here, the resonance and background terms of Eq. (2.8) interfere ; but if the soft-cylinder background is coherently subtracted from S_n^{pp}, one obtains the pure resonances of Fig. 3.1 (b). One notices that peaks of the same order j recur in successive modal amplitudes at successively higher frequencies ; they represent "Regge recurrences" of the jth surface wave spanning the cavity's circumference successively with n = 1,2,3...wavelengths. The corresponding phase velocity dispersion curves, Eq. (2.10a), are shown in Fig. 3.2.

The background terms $S_n^{(o)ik}$ for an empty cavity themselves have resonances, but with large imaginary parts so that e.g. curves $|S_n^{(o)pp} - 1|$ appear smooth, as the broad lobes underlying the curves of Fig. 3.1(a). One may find their frequency resonances, however, by a complex pole search, as shown in Fig. 3.3 for a spherical void in steel [13]. The poles in the $\alpha \equiv k_p a$ plane clearly fall into layers due to Rayleigh (R), compressionnal (P), and shear (S) type (external) surface waves ; their dispersion curves are shown in Fig. 3.4. Comparing this with Fig. 3.2, one sees that dispersion curves for external surface waves start from the origin and reach a plateau for high frequencies, while for internal surface waves, they start at infinity from a low-frequency cutoff, whence they descend to a plateau. This can be understood geometrically.

For a solid inclusion in a solid, results are shown in Figs. 3.5 and 3.6. If the total scattering amplitude, e.g. the modulus of Eq. (2.3), is plotted vs. frequency, a rather smooth curve results (solid curve in Fig. 3.5 [22]).

If, however, one plots individual modes n in the fashion of Fig. 3.1(b), peaks appear (depending, of course, on the materials), which may be used to obtain dispersion curves, Fig. 3.6. These indicate the existence of two types of surface waves (solid and dashed), probably also a Rayleigh wave (lowest dashed curve).

4. EXPERIMENTS.

Experimental results on ultrasonic wave scattering from cavities and inclusions were obtained both in the frequency domain and in the time domain (short pulses) [20 - 23]. Pulse experiments obtained geometrical and surface wave paths from arrival times. Besides spherical and cylindrical shapes, spheroids have also been investigated. The broken curve in Fig. 3.5 shows backscattering results for a spherical tungsten carbide inclusion in titanium [22], as compared to theory.

5. DISCUSSION.

The resonance scattering theory (RST) as developed for acoustic scattering from solids, has here been applied to ultrasonic scattering from cavities and inclusions. Internal resonances appear prominently if the properties (e.g. density) of cavity material and its filler differ substantially ; if this is not the case, they are less evident. External resonances on cavities are always hard to see ; but in all cases (internal and external), resonance frequencies in the complex plane can be calculated no matter how large their imaginary part, and their layers define clearly the various types of surface waves. Experiments have not yet been designed specifically to detect resonances; in a sense, the pulse arrivals indicate these indirectly, being a coherent sum of a number of resonances [17].

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FIGURE CAPTIONS

Fig. 3.1. Modal resonances for a water-filled cylindrical cavity in aluminum : (a) $|S_n^{pp}-1|$ plotted vs. $k_p a$,
(b) $|S_n^{pp} - S_n^{(o)pp}-1|$, background subtracted.

Fig. 3.2. Dispersion curves of internal surface waves in a water-filled cylindrical cavity in aluminum.

Fig. 3.3. Complex-frequency poles of scattering amplitudes from a spherical void in steel, showing resonances due to Rayleigh (R), compressional (P) and shear (S) type surface waves.

Fig. 3.4. Dispersion curves of surface waves whose resonances furnish the poles in Fig. 3.3.

Fig. 3.5. Theoretical (solid) and experimental (broken curve)backscattering from a spherical tungsten carbide inclusion in titanium.

Fig. 3.6. Dispersion curves for two families of surface waves on an iron sphere in aluminum.

Fig. 3.1a

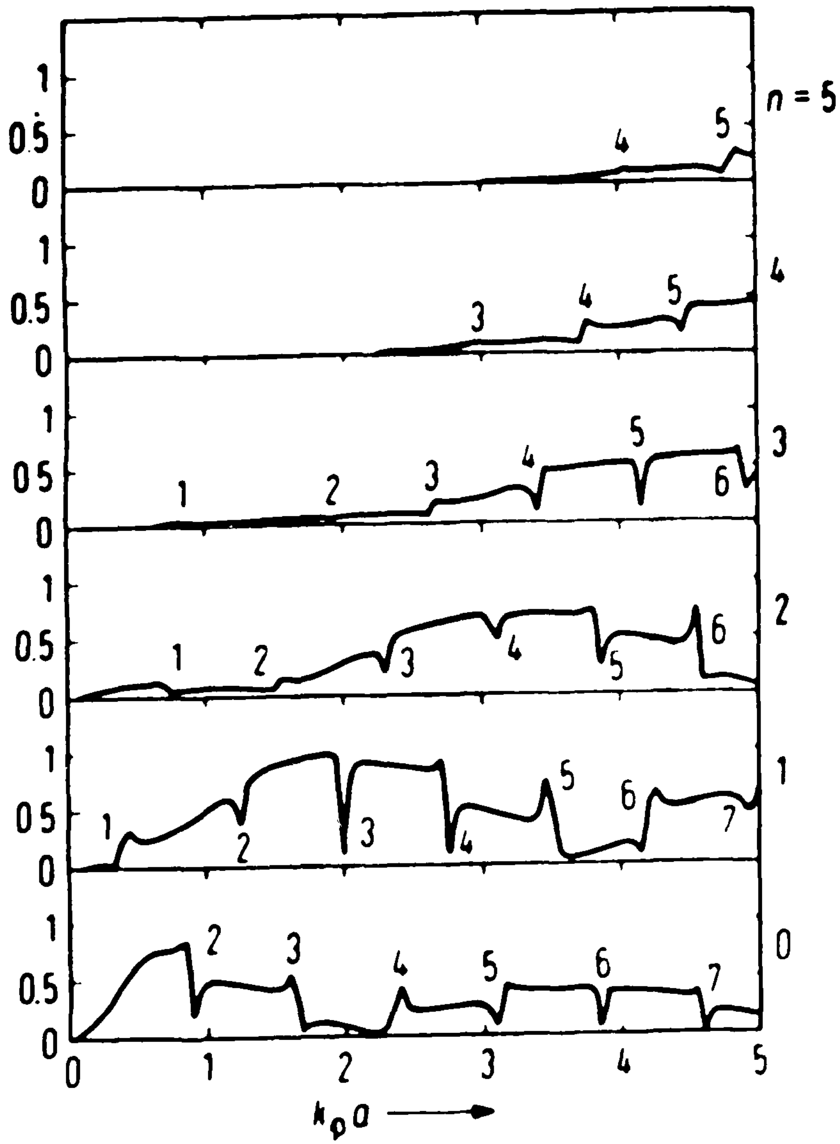


Fig. 3.1b

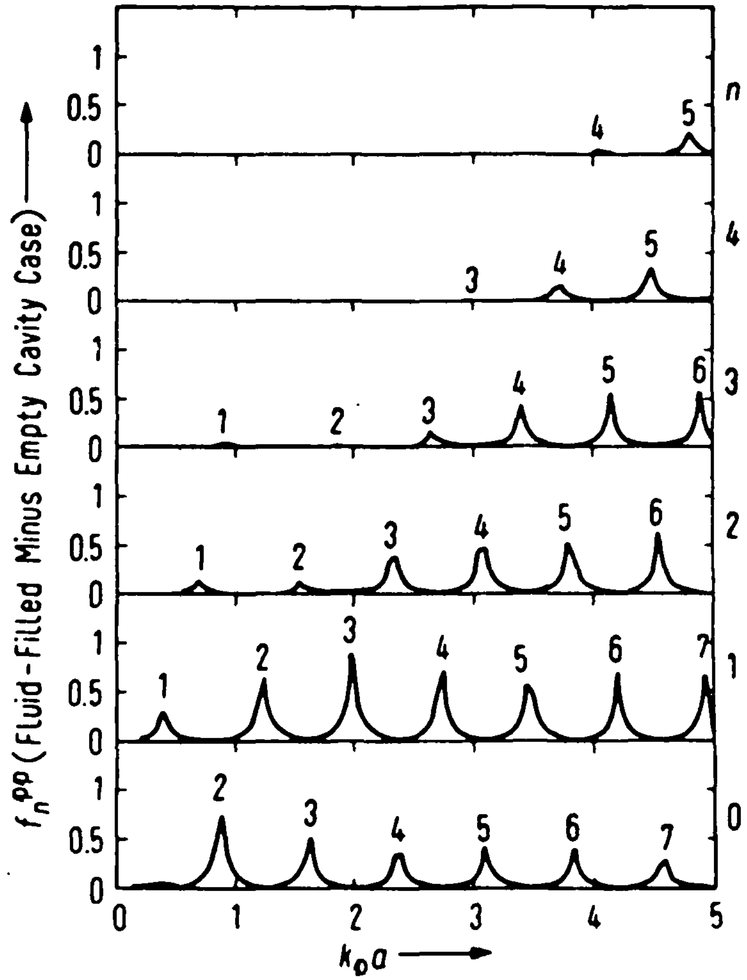
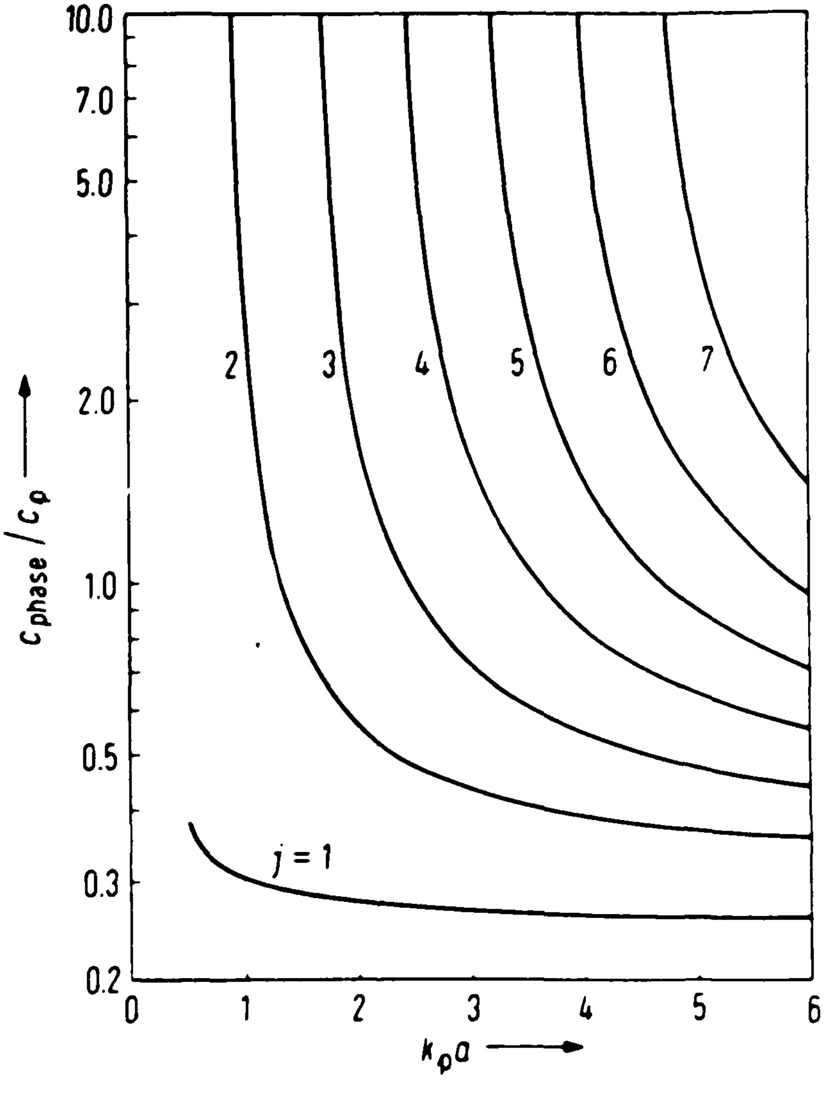


Fig. 3.2



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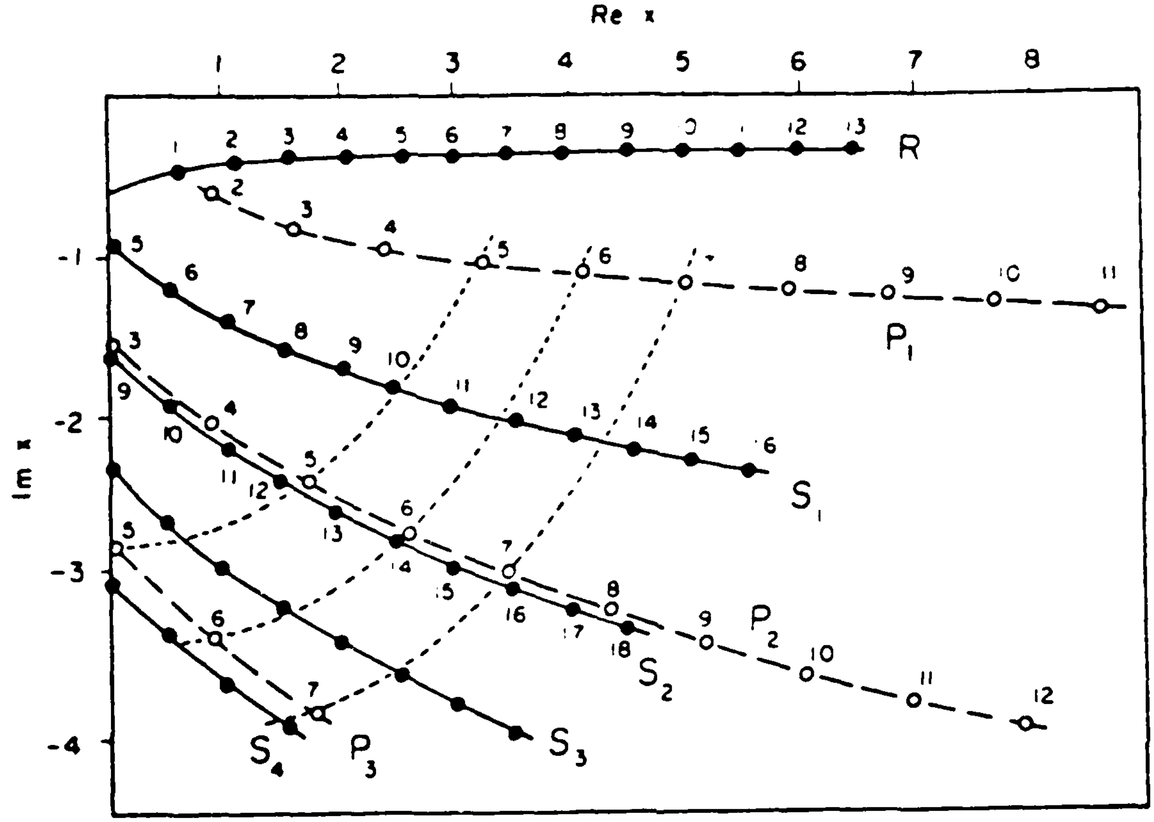
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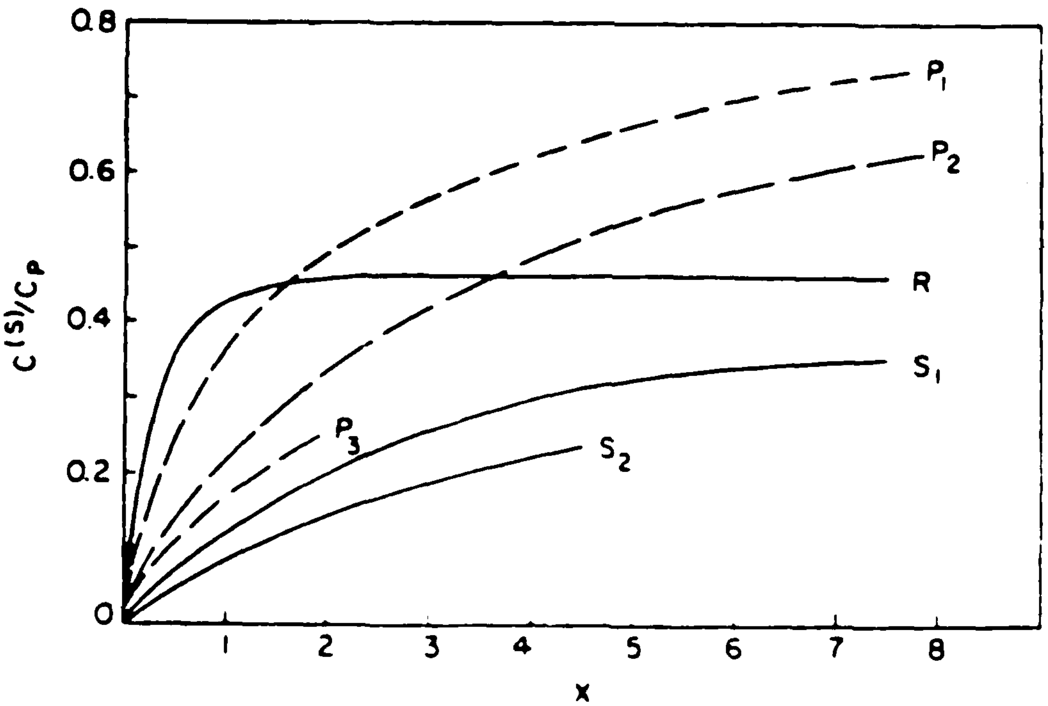
S_3
S_4
P_3

Fig. 3.4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0.2
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2
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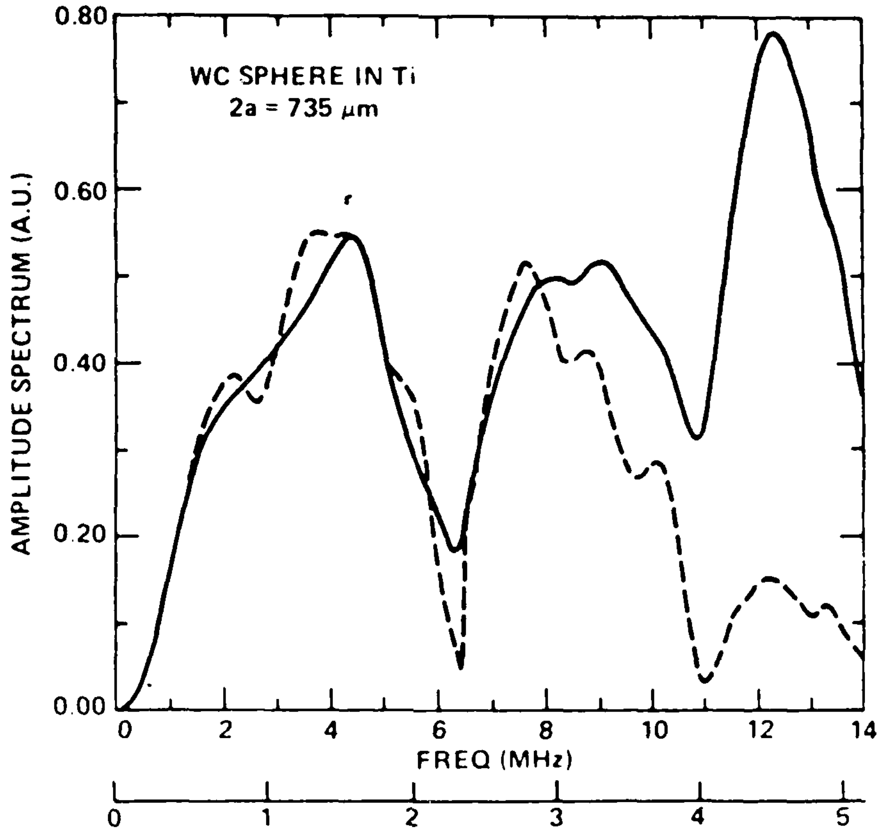
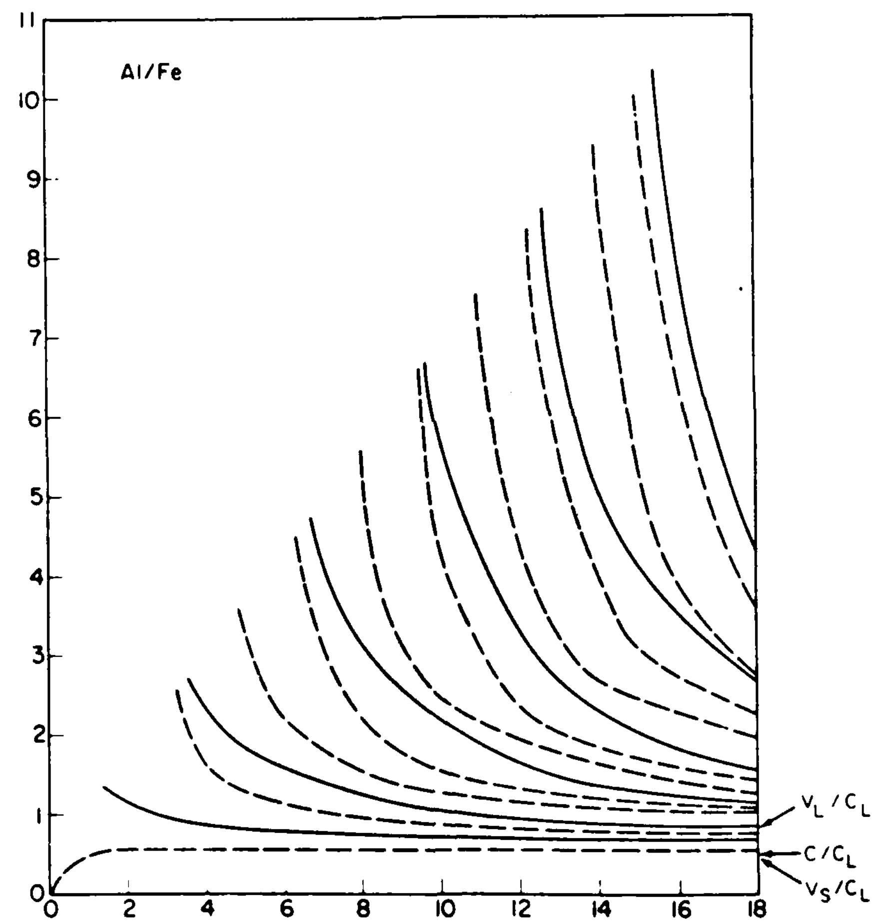
8
x
P_1
P_2
R
S_1
P_3
S_2

Fig. 3.5



WC SPHERE IN Ti
2a = 735 μm
AMPLITUDE SPECTRUM (A.U.)
0.80
0.60
0.40
0.20
0.00
0
2
4
6
8
10
12
14
FREQ (MHz)
0
1
2
3
4
5

Fig. 3.6



DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

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SCATTERING FROM INHOMOGENEITIES

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SUMMARY : A microinhomogeneous medium, consisting of randomly distributed cavities or inclusions in a homogeneous elastic matrix, can be represented as a dispersive homogeneous medium with effective material constants (moduli, bulk wave speeds, and absorptions). For wavelengths long compared to the size of the scatterers, Kuster and Toksöz have developed a method (not including rescattering) which obtains these effective material properties by comparing exact and effective monopole, dipole and quadrupole amplitudes. We extend this approach to the case where the wavelength is comparable to the size of the scatterers (assumed spherical) ; in this case, particle resonances are taken into account and lead to widened resonances in the effective material parameters. The cases of bubbly liquids, of perforated solids, and of solids with solid inclusions (particulate composites) are treated in this fashion. Measurements by Kinra and Anand verify our results. In addition, many previous results for the effective moduli of composites, obtained in the static (i.e., low-frequency) limit, are recovered as particular cases of our approach.

RESUME : Un milieu micro-inhomogène, qui consiste en cavités ou inclusions distribuées dans une matrice élastique homogène de façon aléatoire, peut être considéré comme un milieu homogène et dispersif possédant des constantes de matériaux équivalentes ou effectives (modules, vitesses des ondes de volume et constantes d'absorption). Dans le cas de longueurs d'onde grandes devant la dimension des objets diffuseurs, Kuster et Toksöz ont développé une méthode (ne prenant pas en compte les effets de diffusion multiple) qui donne les propriétés des matériaux équivalents en comparant les amplitudes monopolaire , dipolaire et quadripolaire exactes et effectives.

Nous avons étendu cette approche au cas où la longueur d'onde devient comparable aux dimensions des objets diffusants (considéré comme sphérique) ; dans ce cas, on tient compte des résonances des particules qui causent des résonances élargies dans les paramètres effectifs des matériaux. On traite de cette façon les liquides contenant des bulles, les solides perforés, et les solides contenant des inclusions. Nos résultats sont vérifiés par les mesures de Kinra et Anand. De plus, des résultats antérieurs concernant les modules effectifs de milieux composés, obtenus dans la limite statique (ou de basse fréquence), sont trouvés ici comme cas particuliers de notre approche.

1. INTRODUCTION.

We employ a method first suggested by Ament [1], and later elaborated on and applied to the case of seismic-wave propagation by Kuster and Toksöz [2], in order to obtain effective material constants (moduli, wave speeds, attenuations) for microinhomogenéous media consisting of a random distribution of spherical solid inclusions, fluid-filled cavities or gas bubbles in an otherwise homogeneous solid or fluid host medium. Our method is based on the scattering of waves, and by a comparison of monopole, dipole and quadrupole scattering amplitudes from the microinhomogeneities with those from a sphere of the effective material, we were able to obtain frequency-dependent effective moduli, wave speeds and attenuations which, in contrast to previous work, contain the effects of monopole resonances of the individual inclusions, as retained by us in the long-wavelength expansion. Our theory neglects rescattering effects and is thus restricted to small to moderate volume concentrations Φ (< 40% in some experiments). By its dynamic nature, it furnishes dispersion curves for wave propagation constants which reproduce well their acoustical branch, as obtained experimentally for random glass spheres in an epoxy matrix [3], while the optical branch (dipole resonances) is not included in our results due to our restriction to monopole resonances. We also discuss our results for the case of water containing air bubbles, as well as recent experiments on this topic [4]. Our general dynamic results are shown to reduce to previously obtained static limits for the effective material constants.

2. GENERAL FORMALISM.

In an elastic host medium containing N solid spherical inclusions, the total compressional-wave potential generated by an incident p-wave is

$$\phi(\vec{r}) = \phi_{inc}(\vec{r}) + \sum_{k=1}^{N} \phi_{sc}^{(k)}(\vec{r}, \vec{r}_k), \tag{2.1}$$

if one neglects multiple scattering and retains only single-scattering amplitudes $\phi_{sc}^{(k)}(\vec{r}, \vec{r}_k)$ from the N scatterers of radii a_k, their centers being at random positions $\vec{r}_k$. Alternately, one may consider a large sphere of radius R, with center at $\vec{r}_o$, consisting of an "effective" medium (quantities denoted by a tilde) and imbedded in the same host material ; it leads to a total field

$$\phi(\vec{r}) = \phi_{inc}(\vec{r}) + \tilde{\phi}_{sc}(\vec{r}, \vec{r}_o). \tag{2.2}$$

We define the effective medium to be such as to produce the same scattered far field as the actual composite, so that at $r \gg R$,

$$\tilde{\phi}_{sc}(\vec{r}, \vec{r}_o) = \sum_{k=1}^{N} \phi_{sc}^{(k)}(\vec{r}, \vec{r}_k). \tag{2.3}$$

In this latter relation, a comparison of monopole (A_o), dipole (A_2) and quadrupole (A_3) scattering amplitudes (which can be thought of as the leading terms in a long-wavelength expansion of ϕ_{sc}) is shown to be sufficient to determine the effective material constants [2]. It is necessary, however, to carry out a similar long-wavelength expansion in the known [5,6] multipole amplitudes A_n (which would appear to be called for here, for the sake of consistency) with a certain amount of care : some terms in A_n contain the square of the frequency, which is normally neglected in the long-wavelength expansion, but multiplied by ρ_1/ρ_2, the ratio of host (1)-to-inclusion (2) densities, which may be large so that the corresponding term is no longer negligible. Retaining such terms in A_o is seen [7,8] to effect an inclusion of monopole resonance terms in the scattering amplitude, which indeed appear prominently in such quantities as the dynamic (frequency-dependent) sound speeds and attenuations in bubbly liquids [7], or in the effective wave speeds and moduli of materials containing fluid-filled cavities [8] or solid inclusions [9]. These resonance

terms are retained here in A_o, but not in A_1 or A_2. For the case of solid inclusions, this leads to the explanation of the acoustical branches of measured dispersion curves [2] for the effective medium, but not their optical branches ; these would have to be explained by retaining resonance terms in A_1.

Denoting, for a sphere of radius R, the normalized frequency $x_{d1} = k_{d1}R$ where $k_{d1} = \omega/c_d$ is the p-wave number in medium 1, one has [8]

$$A_o = \frac{x_{d1}^3}{3i} \frac{3 - x_{of}^2 - x_{os}^2 + x_{oi}^2}{x_{d1}^2(1 + i x_{d1}) - x_{of}^2 - x_{os}^2 + x_{oi}^2} , \tag{2.4a}$$

$$A_1 = \frac{x_{d1}^3}{9i}\left(1 - \frac{\rho_2}{\rho_1}\right) , \tag{2.4b}$$

$$A_2 = -\frac{x_{d1}^3}{3i} \frac{4\mu_1(\mu_2 - \mu_1)}{6\mu_2(k_{e1} + 2\mu_1) + \mu_1(9k_{e1} + 8\mu_1)} , \tag{2.4c}$$

where

$$x_{of}^2 = (3\rho_2/\rho_1)(c_{d2}/c_{d1})^2 \tag{2.5a}$$

is the Minnaert [10] bubble resonance frequency,

$$x_{os}^2 = (2c_{s1}/c_{d1})^2 \tag{2.5b}$$

that of the Meyer-Brendel-Tamm [11] resonance for evacuated cavities in solids, and

$$x_{oi}^2 = (\rho_2/\rho_1)(2c_{s2}/c_{d1})^2 \tag{2.5c}$$

an additional resonance due to shear in the inclusion ;

$$k_{e1} = \lambda_1 + \frac{2}{3}\mu_1 \tag{2.5d}$$

is the elastic bulk modulus of medium 1 (λ and μ being the Lamé constants).

3. SPECIFIC RESULTS

We may now compare A_n ($n = 0, 1, 2$) separately in the fashion of Eq. (2.3). This determines elastic moduli and wave speeds as listed in Ref. [9] . One has, e.g., for the (normalized) effective p-wave speed :

$$\frac{\tilde{c}_2}{c_{d1}} = \frac{1}{\left[1-(1-\rho_2/\rho_1)\Phi\right]^{1/2}}\left(\frac{2(U^2+V^2)}{(X^2+Y^2)^{1/2}(U^2+V^2)^{1/2}+(XU+YV)}\right)^{1/2}, \tag{3.1}$$

where

$$X = 1-\Psi_R, \qquad Y = -\Psi_I, \tag{3.2a}$$

$$U = 1-\frac{4}{3}\left(\frac{c_{s1}}{c_{d1}}\right)^2 X\left(1-\frac{\tilde{\mu}_2}{\mu_1}\right), \tag{3.2b}$$

$$V = -\frac{4}{3}\left(\frac{c_{s1}}{c_{d1}}\right)^2 Y\left(1-\frac{\tilde{\mu}_2}{\mu_1}\right), \tag{3.2c}$$

$$\frac{\tilde{\mu}_2}{\mu_1} = \left[1+\frac{6\mu_2}{\mu_1}\,\frac{k_{e1}+2\mu_1}{9k_{e1}+8\mu_1}-\left(1-\frac{\mu_2}{\mu_1}\right)\Phi\right]\times$$
$$\times\left[1+\frac{6\mu_2}{\mu_1}\,\frac{k_{e1}+2\mu_1}{9k_{e1}+8\mu_1}+\frac{6(k_{e1}+2\mu_1)}{9k_{e1}+8\mu_1}\left(1-\frac{\mu_2}{\mu_1}\right)\Phi\right]^{-1}; \tag{3.3}$$

$$\Psi_R = \frac{4\pi A}{3}\int_0^\infty \frac{B\,g(a)}{B^2+C^2}\,a^3\,da, \tag{3.4a}$$

$$\Psi_I = \frac{4\pi A}{3}\int_0^\infty \frac{C\,g(a)}{B^2+C^2}\,a^3\,da, \tag{3.4b}$$

$$A = 1-\frac{\rho_1 c_{d1}^2}{\rho_2 c_{d2}^2}+\frac{4}{3}\,\frac{\rho_1 c_{s1}^2}{\rho_2 c_{d2}^2}-\frac{4}{3}\,\frac{\rho_2 c_{s2}^2}{\rho_2 c_{d2}^2}, \tag{3.5a}$$

$$\mathcal{B} = 1 + \frac{4}{3}\frac{\rho_1 c_{s1}^2}{\rho_2 c_{d2}^2} - \frac{1}{3}\frac{\rho_1 c_{d1}^2}{\rho_2 c_{d2}^2}\left(\frac{2\pi a f}{c_{d1}}\right)^2 - \frac{4}{3}\frac{\rho_2 c_{s2}^2}{\rho_2 c_{d2}^2}, \tag{3.5b}$$

$$\mathcal{C} = \frac{1}{3}\frac{\rho_1 c_{d1}^2}{\rho_2 c_{d2}^2}\left(\frac{2\pi a f}{c_{d1}}\right)^2. \tag{3.5c}$$

Here, the volume concentration is

$$\Phi = \frac{4\pi}{3}\int_0^\infty g(a)\, a^3\, da, \tag{3.6a}$$

where g(a) is the radius distribution of the inclusions. From

$$\tilde{\rho}_2 = (1-\Phi)\,\rho_1 + \Phi\,\rho_2 \tag{3.7}$$

and Eq. (3.3), one finds the effective s-wave speed :

$$\tilde{c}_{s2} = (\tilde{\mu}_2/\tilde{\rho}_2)^{1/2}, \tag{3.8}$$

which is frequency independent.

Figure 3.1 shows theoretical [9] and measured [3] results for the frequency dependence of the effective p-wave speed in a medium of glass spheres (average radius 1 mm) in epoxy, at concentrations Φ = 5% and 15%. Figure 3.2a shows this speed at f = 0.5 MHz plotted vs. Φ, and Fig. 3.2b displays the shear speed vs. Φ ; the agreement is good up to Φ = 40%.

4. PARTICULAR CASES

The static effective material constants are obtained in the limit $f \to 0$. Our above results then agree with the previous static results of Mal and Bose [12] , Kerner [13] ,and Hashin [14]. For rigid inclusions (λ_2, $\mu_2 \to \infty$) of dilute concentration ($\Phi \ll 1$) we recover the results of Moon and Mao [15], and if the host medium is additionally assumed incompressible (Poisson ratio

$\nu \cong \frac{1}{2}$, one has

$$\tilde{\mu}_2 / \mu_1 = 1 + \frac{5}{2} \Phi , \tag{4.1}$$

a result obtained by Einstein [16].

5. GAS BUBBLES IN FLUIDS

Here one has $c_{s1} = c_{s2} = 0$, further U = 1, V = 0. The theory then reduces to a previously treated case [7], and Fig. 5.1a presents numerical results for water containing a = 1 mm air bubbles at $\Phi = 0.000377$ (their resonance frequency being 3.285 kHz) compared to experimental results of Silberman [17] : the top portion shows absorption, the botton portion the effective sound speed. Figure 5.1b gives more recent measurements (crosses) at $\Phi = 0.0000267$ for a bubble distribution centered on a = 45 μm, compared to theory (solid curves) [4]. In this latter work, it is also shown how, with one measurement of c at low frequency, and two at high frequency, the first three moments of the bubble size distributions can be experimentally determined.

6. CONCLUSION

The foregoing theory permits the determination of dynamic effective material constants for microinhomogeneous media containing gas bubbles, fluid-filled cavities or solid inclusions (treated as spherical), for low to moder concentrations of the inhomogeneities, and taking their resonanc as functions of frequency) into account. For higher concentrations, self-consistent methods have to be invoked such as that of Chaban [18] ; this latter method, however, treats resonances in an ad-hoc fashion only.

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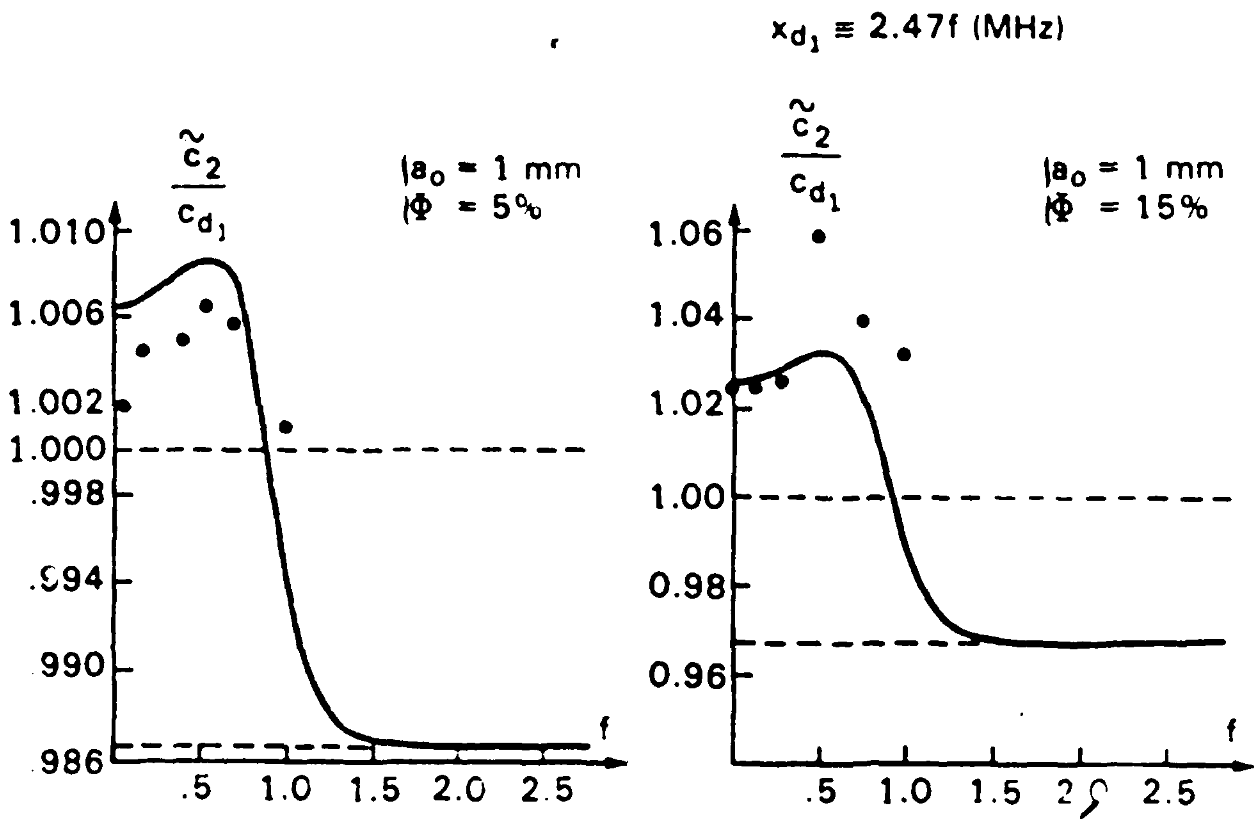
FIGURE CAPTIONS

Fig. 3.1. Effective sound speed $\tilde{c}_2$ vs. frequency (0-2.5 MHz) for a = 1 mm glass spheres in epoxy (concentrations Φ = 5% and 15%).

Fig. 3.2. Effective wave speeds for a = 1 mm glass spheres in epoxy, vs. concentration Φ : (a) effective sound speed $\tilde{c}_2$ at f = 0.25 MHz, (b) effective shear wave speed.

Fig. 5.1. Attenuation (top) and effective sound speed (bottom) in bubbly water at (a) a = 1 mm, Φ = 0.000377, (b) $\bar{a}$ = 45 μm, Φ = 0.0000267.

Fig. 3.1



$x_{d_1} \equiv 2.47f$ (MHz)
$\frac{\tilde{c}_2}{c_{d_1}}$
$a_o = 1$ mm
$\Phi = 5\%$
1.010
1.006
1.002
1.000
.998
.994
.990
.986
.5
1.0
1.5
2.0
2.5
f
$\frac{\tilde{c}_2}{c_{d_1}}$
$a_o = 1$ mm
$\Phi = 15\%$
1.06
1.04
1.02
1.00
0.98
0.96
.5
1.0
1.5
2.0
2.5
f

Fig. 3.2a

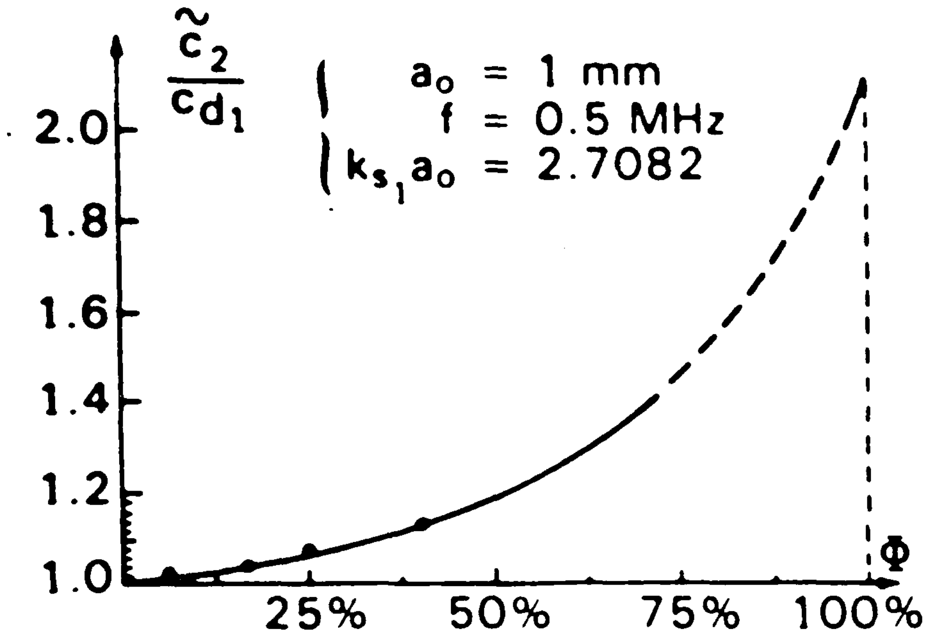


Fig. 3.2b

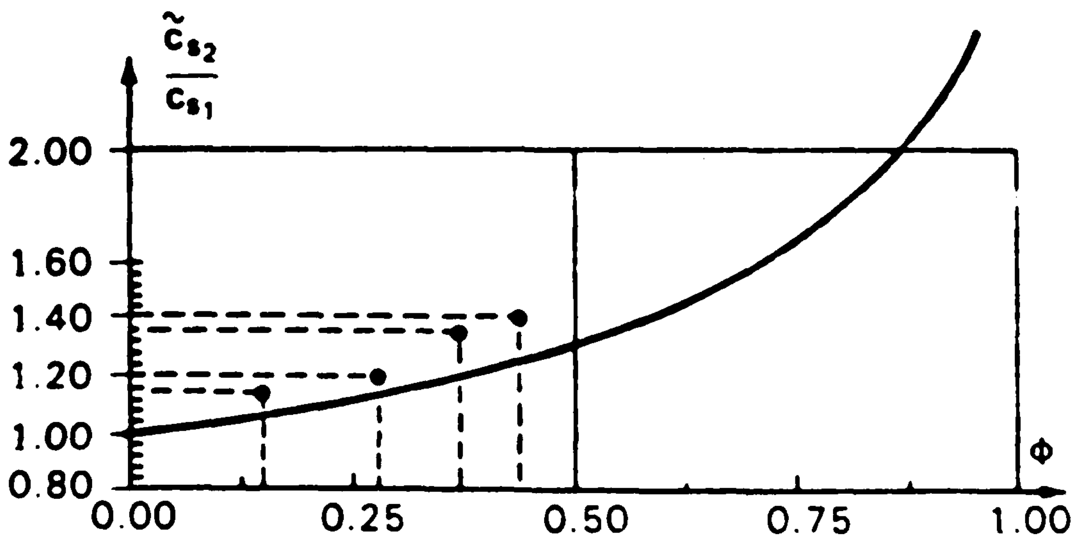
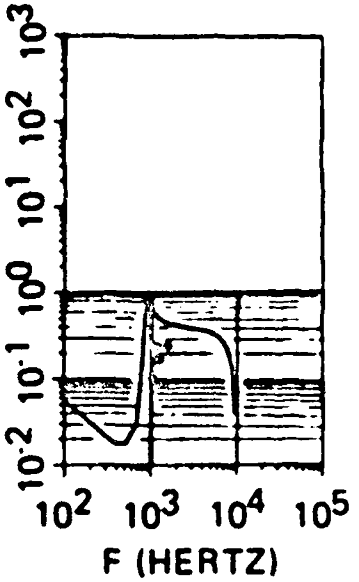


Fig. 5.1a


10^3
10^2
10^1
10^0
10^-1
10^-2
10^2 10^3 10^4 10^5
F (HERTZ)

(c)

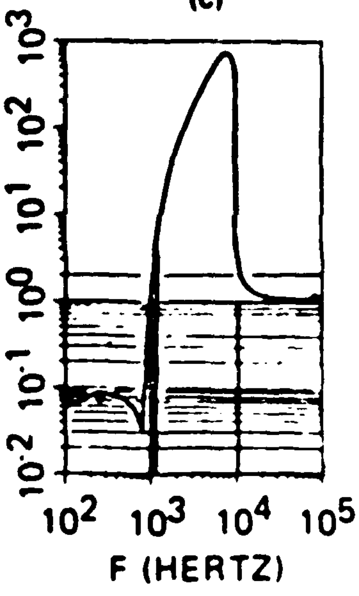

10^3
10^2
10^1
10^0
10^-1
10^-2
10^2 10^3 10^4 10^5
F (HERTZ)

Fig. 5.1b (top)

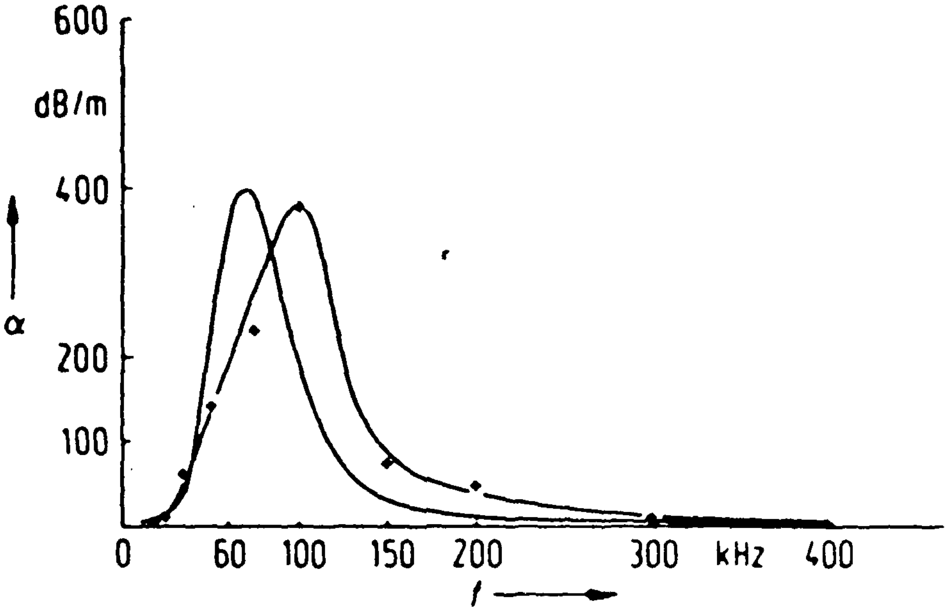
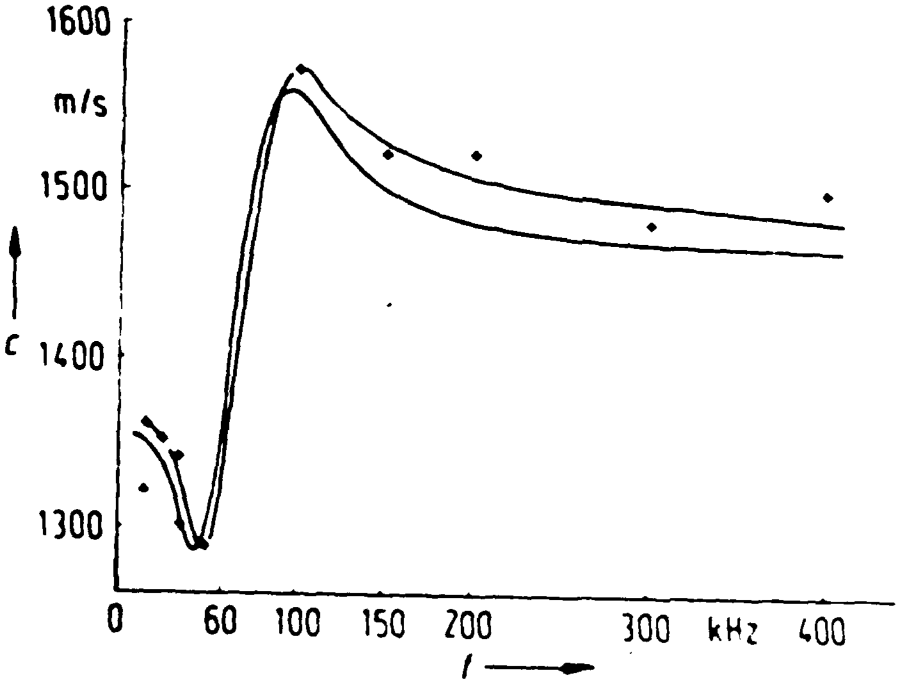


Fig. 5.1b (bottom)



DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

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SCATTERING OF SHORT AND LONG SOUND PULSES : CONNECTION WITH THE SINGULARITY EXPANSION METHOD.

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SUMMARY : Baum's "Singularity Expansion Method" (SEM), formulated for radar scattering, is based on the observation that transient scattered echoes appear to be composed of a sum of decaying sinusoids. Fourier-transforming these expressions into frequency space reveals a manifold of simple poles in the complex frequency plane, identical with those of the Resonance Scattering Theory (RST), and commonly grouped in layers. We have carried out a time dependent analysis of scattering from rigid and elastic spheres and cylinders, with the following results. The use of short pulses (of spatial extent small compared to the scatterer's dimension) corresponds to the echo being a residue sum over a large number of SEM poles, and the residue sum over one given pole layer produces a sequency of echo pulses corresponding to a creeping wave repeatedly circumnavigating the sphere with the appropriate group velocity. The use of finite-length sinusoidal wave trains (long compared to the scatterer's extension) produces a reflected wave train, coherently superimposed by a sequence of overlapping creeping wave trains, which cause initial transients as well as a final transient tail following the echo. These transients only appear if the carrier frequency coincides with an eigenfrequency of the target, and the tail amplitude plotted as a function of frequency then reproduces the spectrum of resonances including their widths, leading to a direct target spectroscopy as accomplished experimentally by Ripoche et al. This tail corresponds to the ringing of a given eigenvibration, which is selectively excited when overlapped by the narrow spectrum of the long incident pulse.

RESUME : La "méthode de développement en singularités" ["Singularity Expansion Method" (SEM)] de Baum, formulée pour le cas de la diffusion radar, est basée sur l'observation que les échos de la diffusion transitoire paraissent être composés d'une somme de sinusoïdes amorties. La transformation de ces sinusoïdes dans l'espace des fréquences par la transformation de Fourier, révèle une multitude de pôles simples dans le plan complexe des fréquences, identiques à ceux de la "Théorie de la diffusion résonnante" ["Resonance Scattering Theory" (RST)], et généralement groupés en couches. Nous avons exécuté une analyse de la diffusion transitoire par des sphères et des cylindres impénétrables ou élastiques, avec les résultats suivants. L'utilisation d'impulsions courtes (d'une extension spatiale petite en comparaison avec les dimensions de l'objet diffusant) correspond au fait que l'écho est une somme sur les résidus d'un grand nombre de pôles SEM, et la somme sur les résidus d'une couche de pôles donnée produit une suite d'impulsions qui correspond à une onde circonférentielle encerclant la sphère plusieurs tours avec la vitesse de groupe appropriée. L'utilisation de trains d'ondes sinusoïdales d'une durée finie (d'une extension spatiale grande en comparaison avec l'étendue de l'objet diffusant) produit un train d'ondes réfléchies, sur lequel est superposée d'une façon cohérente une suite de trains d'ondes circonférentielles qui se chevauchent et causent des transitoires initiaux, ainsi qu'une traînée transitoire qui suit l'écho. Ces transitoires apparaissent seulement si la fréquence porteuse du train d'ondes incident coïncide avec une fréquence propre de la cible : le tracé de la courbe donnant l'amplitude de la traînée en fonction de la fréquence est alors identique à celui des résonances, avec leur largeur, ce qui autorise une spectroscopie directe de la cible, ainsi que l'ont montré Rinoche et al expérimentalement. La traînée correspond à la réémission libre d'une vibration propre distincte, excitée d'une façon sélective si sa fréquence se trouve dans la région étroite du spectre de l'impulsion (longue) incidente.

1. INTRODUCTION.

The singularity Expansion Method (SEM), first formulated for the case of radar scattering [1], is based on the observation that the echoes of radar pulses scattered from a finite target appear as the superposition of damped sinusoids,

$$f_{sc}(t) = \sum_{\alpha=1}^{N} R_\alpha e^{\hat{s}_\alpha t}, \tag{1.1}$$

with complex amplitudes R_α and exponents $\hat{s}_\alpha$. Taking the Laplace transform, this pulse shape then indicates the presence of complex-frequency poles in the scattering amplitude,

$$\tilde{f}_{sc}(s) = \sum_{\alpha=1}^{N} \frac{R_\alpha}{s - \hat{s}_\alpha}, \tag{1.2}$$

where $s = \omega/i$, and $\omega_\alpha = \mathrm{Im}\, s_\alpha$ are the natural frequencies of the target whose existence gives rise to resonances in the scattering amplitude, Eq.(1.2). This latter equation corresponds to a single-frequency excitation of the resonances. If the incident amplitude is pulsed in time, it can be represented by the Fourier integral

$$f_{inc}(z,t) = \int G(k) e^{ik(z-ct)}\, dk/2\pi, \tag{1.3}$$

where $k = \omega/c$. Its spectrum G (k) weighs the factor R_α in Eqs. (1.1), (1.2).

A short pulse is characterized by a wide spectrum G(k) ; for $G(k) \equiv 1$, one has $f_{inc}(z,t) = \delta(z-ct)$. Numerous short-pulse experiments for acoustic scattering from submerged elastic bodies [2-6], or for elastic-waves scattering from cavities [7,8] have shown that the echo then consists of a reflected pulse, with a shape close to that of the incident pulse [9], followed by a sequence of pulses slightly widened as compared to the incident pulse shape [10,11], which correspond to the multiple encirclement of the scatterer by surface (creeping) waves. Theoretically, the poles $\hat{k}_\alpha \equiv i\hat{s}_\alpha/c$ at the scattering amplitude fall into layers in the complex frequency plane, generally parallel to the real axis and displaced from it at successively increasing imaginary spacings. The wide pulse spectrum covers many poles in the first layer and in succeeding ones, and their coherent residue sums add up to the experimentally observed time sequence of pulses [12], one sequence for each layer sum which hence corresponds to a given mode of surface wave. The pulses were found to propagate with the group velocity of the surface waves.

An incident pulse of long duration has a narrow spectrum G(k), whose weight in Eq.(1.2) can radically limit the number of poles contributing to Eq.(1.1). In fact, if the pulse duration is chosen long enough, the

width of G(k) may be made less than the spacing between the real parts of the poles, $k_\alpha \equiv \mathrm{Re}\,\hat{k}_\alpha$. If then the peak of G(k) coincides with the position k_α of the αth pole, Eq.(1.1) becomes

$$f_{sc}(t) \cong R_\alpha\, G(k_\alpha)\, e^{\hat{s}_\alpha t}, \qquad (1.4)$$

having been reduced to the contribution of one single pole α, which appears in the form of one single damped sinusoid with its decay factor $\exp(\mathrm{Re}\,\hat{s}_\alpha)t \equiv \exp(-|\mathrm{Im}\,k_\alpha|)ct$, $\mathrm{Im}\,k_\alpha < 0$, representing the "ringing" of the resonance α ; this ringing is continuous and of duration $(|\mathrm{Im}\,\hat{k}_\alpha|c)^{-1}$. Such a use of long pulses leads to the possibility of exciting target resonances corresponding to individual poles of the scattering amplitude, and hence determining the natural frequencies, the imaginary parts $\mathrm{Im}\,\hat{k}_\alpha$ of the pole positions (from the decay constant of the ringing), and the residues.

The information thus obtained can conceivably be utilized for target identification purposes [13]. The pioneering experimen' of Maze, Ripoche et al. [14-18] have clearly demonstrated the ringing of target resonances as excited by long incident sound pulses, thus obtaining the target's eigenfrequency spectrum in a direct fashion, and opening up the way to what may be called "acoustic spectroscopy" [13,19].

2. SCATTERING OF SHORT PULSES.

The acoustic field in the presence of a sphere is [20]

$$p = \frac{1}{2}\sum_{n=0}^{\infty}(2n+1)i^n\left\{h_n^{(2)}(kr) + S_n h_n^{(1)}(kr)\right\}P_n(\cos\theta), \qquad (2.1)$$

and its poles are given by those of the S-function

$$S_n = S_n^{(r)}\,\frac{F_n^{-1} - z_n^{(2)-1}}{F_n^{-1} - z_n^{(1)-1}}, \qquad S_n^{(r)} = -\frac{h_n^{(2)\prime}(x)}{h_n^{(1)\prime}(x)} \qquad (2.2)$$

(where x = ka, a = sphere radius) in the complex frequency plane. These are shown in Fig. 2.1 [21] for the case of a tungsten carbide sphere.

Figure 2.1(a), top, presents the poles $\hat{x}_{n\ell}$ corresponding to the resonances of external surface waves (close to the series of $h^{(1)'}(x)$; the pole layers corresponding to the ℓ th wave ($\ell = 1,2,\ldots$) are connected by solid lines (while the poles of a given mode n are connected by dashed curves). Part (b) of the figure, bottom, shows the poles due to internal surface waves (given by the zeros of $F_n^{-1} - z_n^{(1)-1}$, the layer labeled $\ell = 1$ corresponding to the Rayleigh wave.

For the case of a rigid sphere ($F_n \to 0$), one obtains by an expansion of the denominator of $S_n^{(r)}$ around its poles $\hat{x}_{n\ell}$, for the ℓth creeping-wave amplitude in the far field, after application of Eq.(1.3) :

$$p_{cr}^{(\ell)}(\theta,\tau) = \frac{1}{r}\sum_{n=0}^{\infty} \frac{n+\frac{1}{2}}{\hat{x}_{n\ell}} \frac{h_n^{(2)'}(\hat{x}_{n\ell})}{h_n^{(1)''}(\hat{x}_{n\ell})} P_n(\cos\theta)\, G\left(\frac{\hat{x}_{n\ell}}{a}\right) e^{-i\hat{x}_{n\ell}\tau}, \tag{2.3}$$

where $\tau = (ct-r)/a$ is a dimensionless time variable. This is the mentioned weighted residue sum over the SEM poles. For an incident δ - pulse (G = 1), Fig. 2.2 shows the backscattered pulse sequence arising from a numerical sum over 50 decaying sinusoids in Eq.(2.3), corresponding to the $\ell = 1$ external creeping wave. The sinusoids are seen to cancel, except at the arrival times of the creeping wave which circumnavigated the sphere over 1/2, 3/2... circumferences, the arrival times being determined by the group velocity of the surface wave.

3. SCATTERING OF LONG PULSES.

This technique was pioneered experimentally by Maze and Ripoche [14-18]. Figure 3.1(a) shows their initial pulse, a rectangular wave train long enough to wrap around their cylindrical target 50-100 times. Figure 3.1(b) presents the reflected echo when the carrier frequency of the train coincides with an eigenfrequency $x_{n\ell} = \mathrm{Re}\hat{x}_{n\ell}$; the figure shows both an initial transient and the pulse's settling down to a steady-state plateau, while after the passage of the reflected train, a tail due to the ringing of the resonance appears. Its decay time determines $\mathrm{Im}\hat{x}_{n\ell}$, and the rate of disappearance of the tail as the carrier frequency is moved away from the resonance frequency determines the width of the resonance. In this way, an eigenfrequency spectrum of the cylinder is obtained as shown in Fig. 1.1(b) of Ref. [13], by performing a sweep of the carrier frequency.

The physics of this ringing has been studied recently both for elastic spheres and cylinders [22]. Figure 3.2 shows an incident wave train (center), its spectrum overlaid on, and coinciding with an interference minimum of the backscattering amplitude for a tungsten carbide sphere (top), and the backscattered signal at resonance ($x_{n\ell} = 14.07$). Figure 3.3 shows similar quantities with the carrier frequency somewhat off resonance ($x = 13,7$). The overlay with the minimum causes a deep constriction in the plateau (steady-state portion) of the reflected signal ; an initial transient descends to the plateau and, after the passage of the reflected pulse, a final transient (tail, $\equiv$ ringing of the resonance) decays in a fashion similar to the initial transient, both showing a pattern of steps first pointed out by Veksler [23].

The transients originate from a coherent superposition of ingredients schematically shown in Fig.3.4. Disregarding the dotted rectangles representing internal reflections in backscattering, one has a specularly reflected wavetrain (large rectangle) and successive circumferential wavetrains (small rectangles). At resonance, all these latter wavetrains interfere constructively with each other and form a tail which consists of steps, hereby approximating the exponential ringing, Eq.(1.4), of the resonance. They also form a coherent initial transient, region (i) of Fig. 3.4, which may interfere either constructively, destructively or in an intermediate fashion with the specular echo depending on the material parameters. In the case of Fig. 3.2, the interference is destructive as indicated by the minimum of the form function ; the interference of the initial transient with the specular echo thus leads to a step-wise descent of the signal towards the quasi-stationary regime [region (ii) of Fig. 3.4], ending with a constricted plateau in Fig. 3.2. For the off-resonance case of Fig. 3.3, the tail has decreased in amplitude (indicating the resonance width), and the interference of the initial transient with the specular pulse has changed.

CONCLUSION.

A complete understanding of the transient scattering of long or short sound pulses from elastic targets is obtained by considering the interplay of the incident pulse spectrum with the SEM pole pattern of target eigenfrequencies in the complex frequency plane, and with the backscattering amplitude as plotted vs. frequency. In the time domain, each pole contributes

a residue in the form of a decaying sinusoid - the ringing of the correspondin scattering resonance. Short echo pulses consist of a sum over many poles, with their ringing all adding up to nonvanishing (short) echo signals of circumferential pulses, with arrival times corresponding to the group velocities of the circumferential waves. Long echo pulses contain a coherent sum of overlapping circumferential wavetrains, constructively interfering at resonance but destructively off resonance, hence leading to an approximately exponentially (actually, step-wise) decaying ringing of the resonance, or to the absence of ringing off resonance. These trains interfere arbitrarily with the specular-echo wave train, causing initial transients and a steady-state plateau in the latter. The success of MIIR [18] as a tool of acoustic spectroscopy [13,19] rests in the direct measurement of resonance amplitudes (via the ringing effect), unencumbered by any interference with the specular-reflection background as in the steady-state regime. Due to the prominence and sharpness of (intrinsic) elastic-wave resonances (i.e., their high Q values), and the ensuing capability of the associated surface waves to encircle the target a large number of times before being attenuated, resonance spectroscopy in underwater acoustics may well become a useful means for target recognition if applied properly and with sufficient insight and care.

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FIGURE CAPTIONS

Fig. 2.1. SEM - pole layers corresponding to the resonances of (a) external (top) and (b) internal (bottom) surface waves on a tungsten carbide sphere in water.

Fig. 2.2. Backscattering pulse sequence of first ($\ell=1$) external circumferential wave on a rigid sphere.

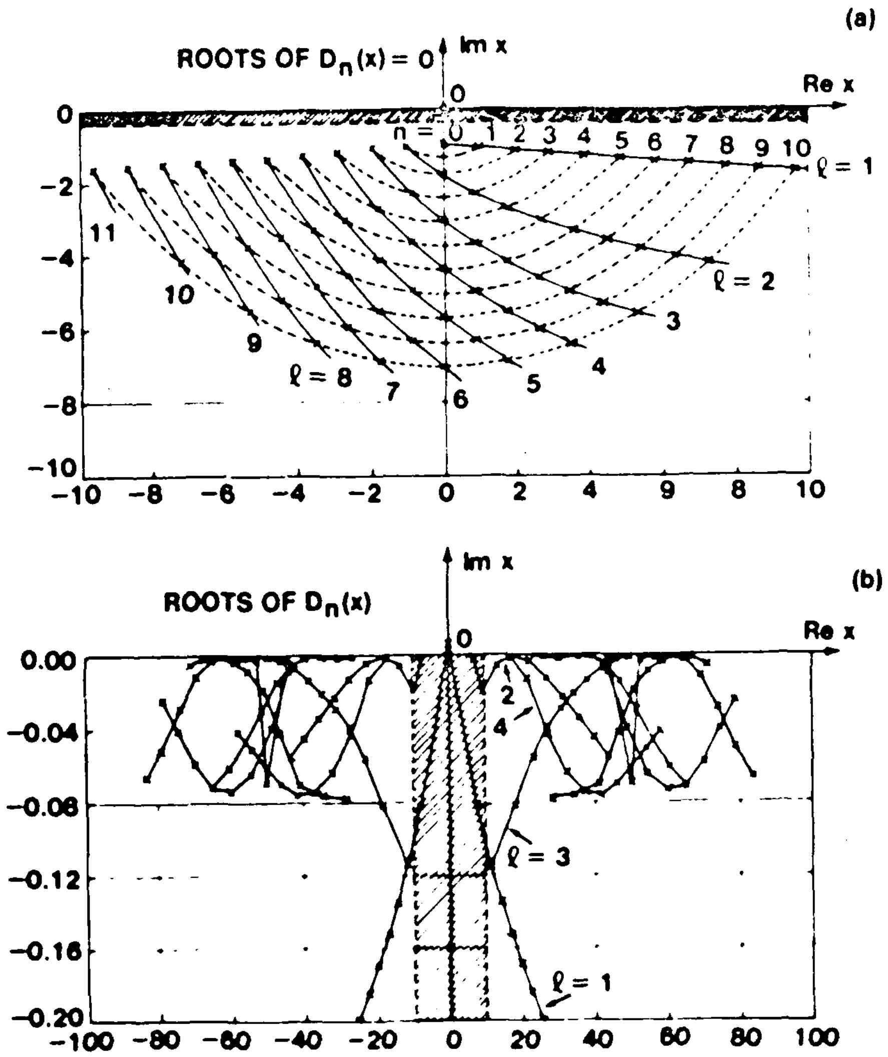
Fig. 3.1. Scattered wave train (a) off-resonance, (b) with its carrier frequency coinciding with a resonance frequency of the target (aluminum cylinder in water), showing initial transient and tail to the ringing of the resonance.

Fig. 3.4. Schematic view of specular (large solid rectangle), penetrating and multiply internally reflected (dotted rectangles), and circumferential wave trains (small solid rectangles), before their coherent addition ; (i) initial transient region, (ii) quasi - steady state region, (iii) final transient region (of ringing).

Fig. 3.2. Amplitude of incident wave train (center), its spectrum overlaid on the backscattering amplitude of a tungsten carbide sphere (top), and backscattered signal (bottom), at a resonance $(x_{n\ell}) = 14.07$).

Fig. 3.3. As in Fig. 3.2, but off resonance ($x = 13.7$).

Fig. 2.1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Re x
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ℓ = 1
ℓ = 2
3
4
5
6
7
ℓ = 8
9
10
11
0
-2
-4
-6
-8
-10
-10 -8 -6 -4 -2 0 2 4 9 8 10
(b)
ROOTS OF $D_n(x)$
Im x
Re x
0
2
4
ℓ = 3
ℓ = 1
0.00
-0.04
-0.08
-0.12
-0.16
-0.20
-100 -80 -60 -40 -20 0 20 40 60 80 100

Fig. 2.2

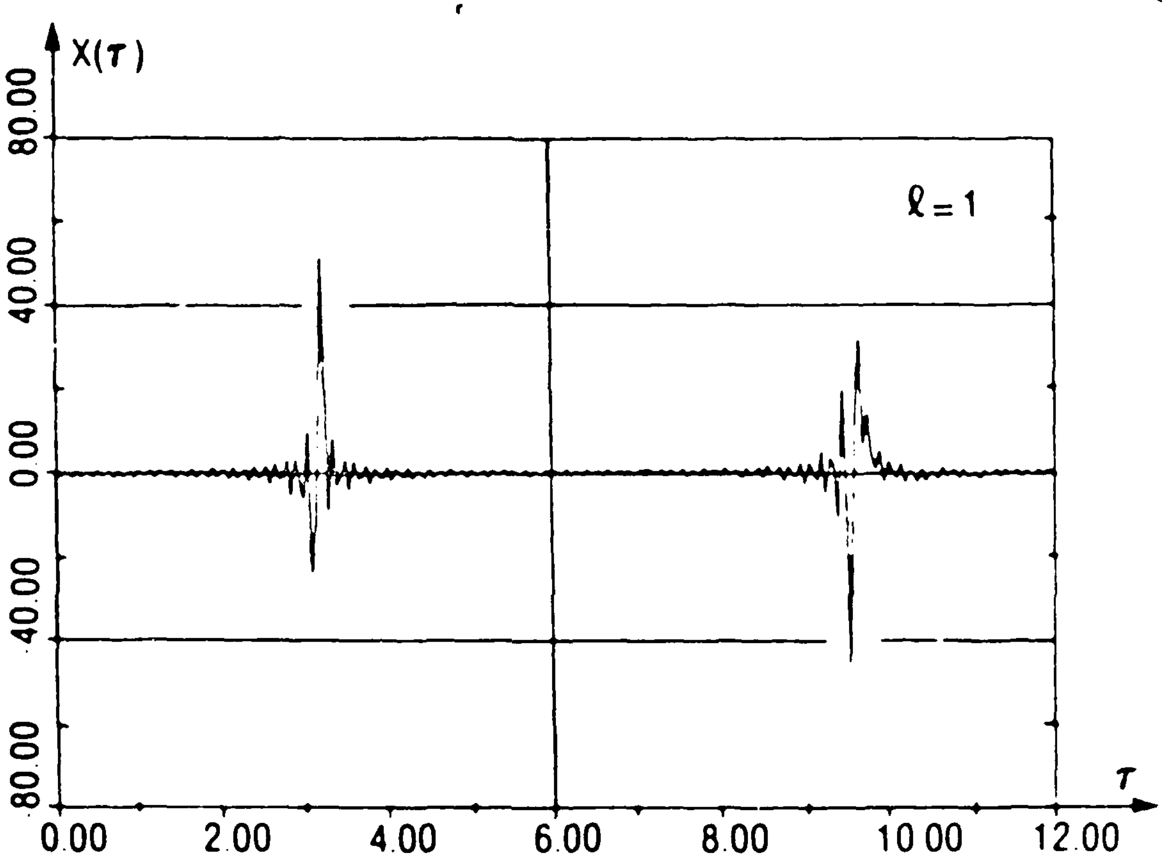
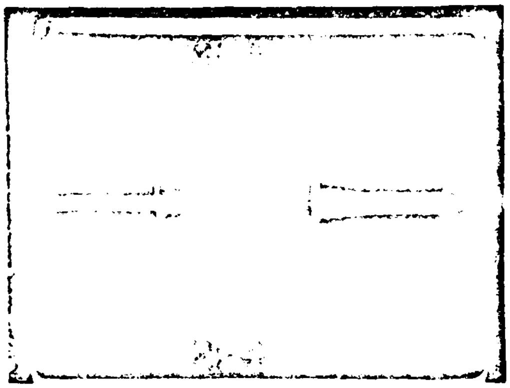
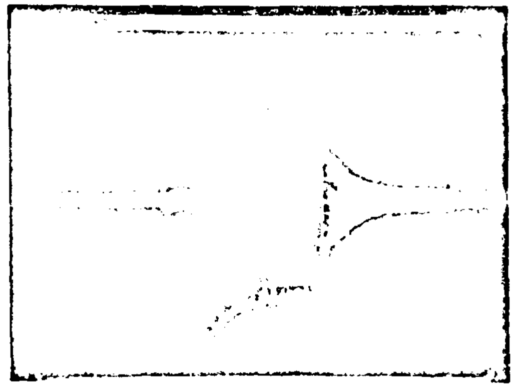


Fig. 3.1a



a)

Fig. 3.1b



b)

Fig. 3.4

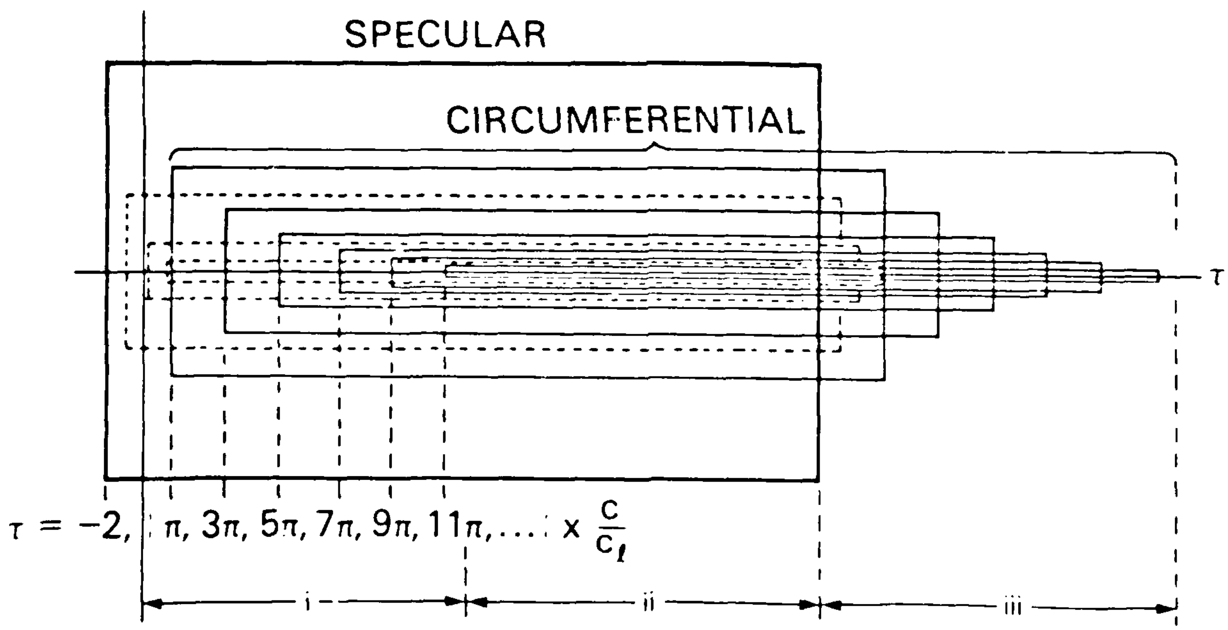
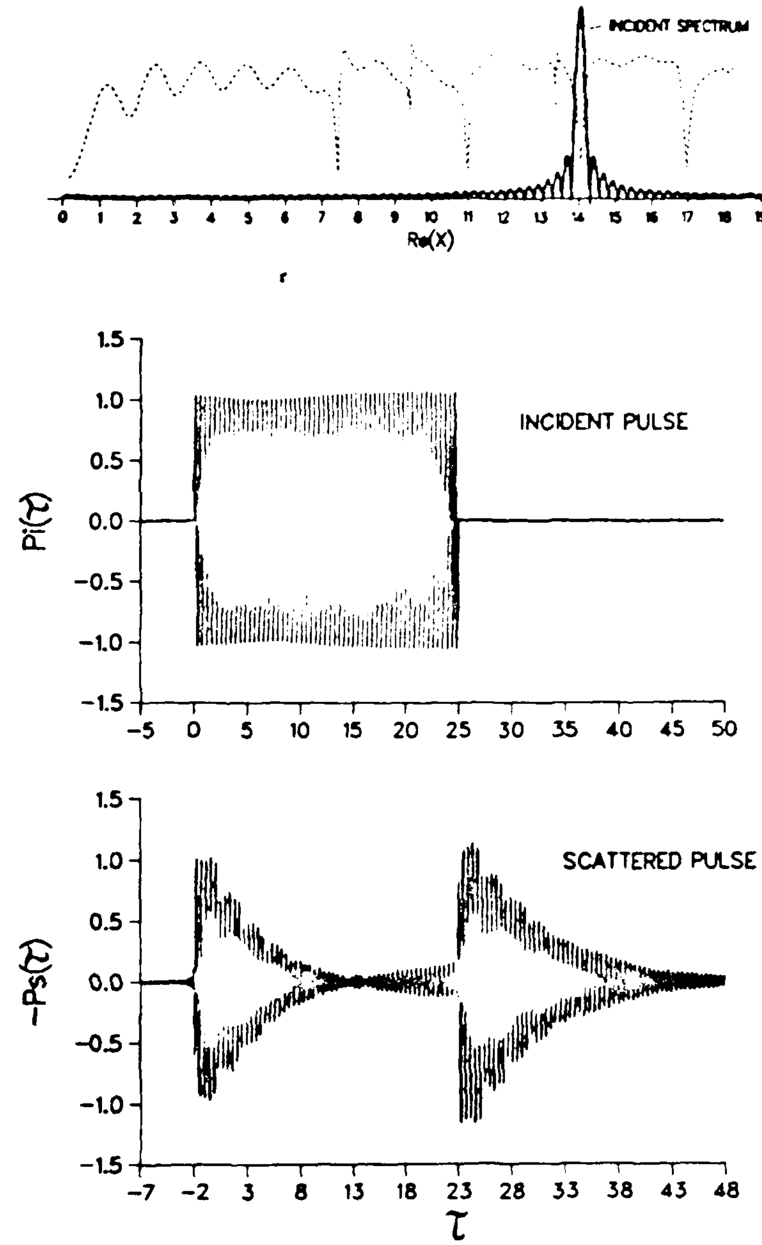


Fig. 3.2



INCIDENT SPECTRUM
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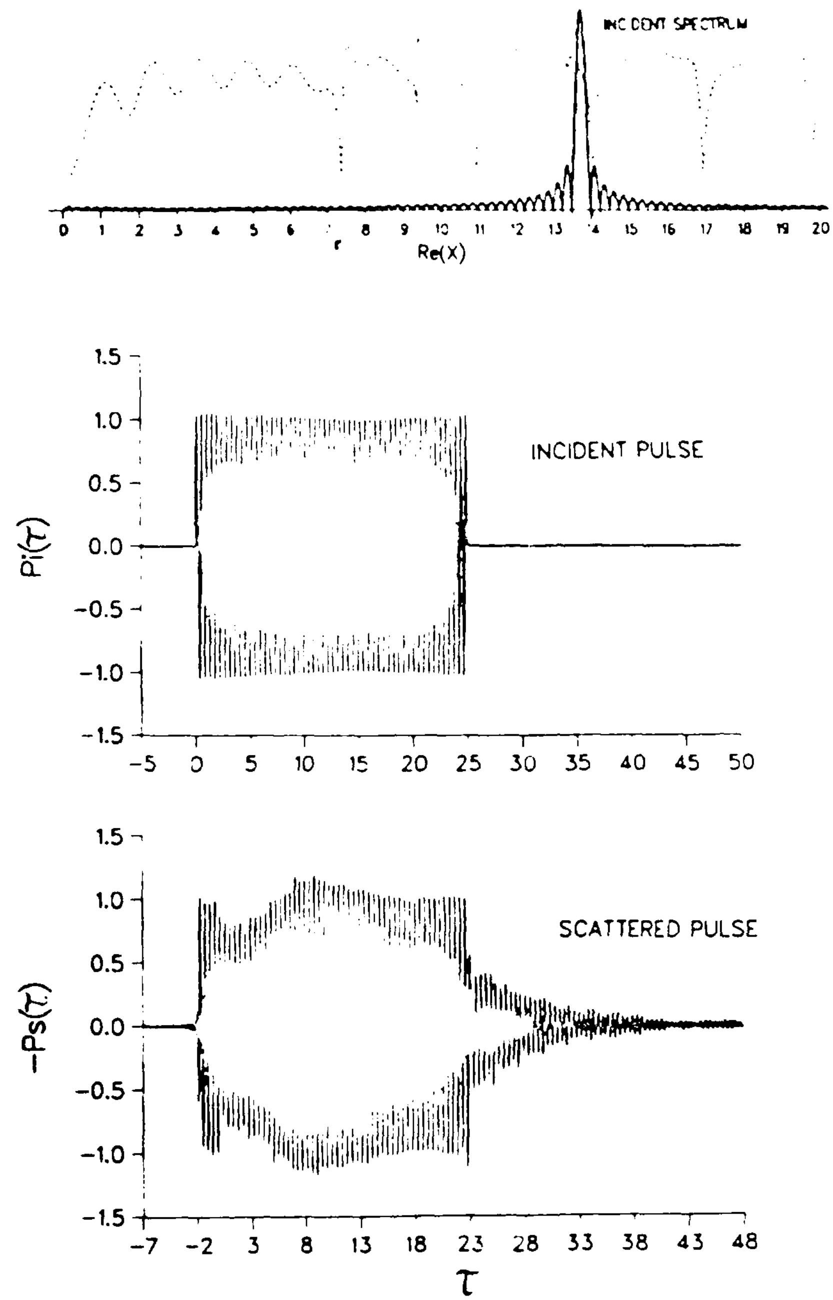
Re(X)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Pi(τ)
INCIDENT PULSE
−5 0 5 10 15 20 25 30 35 40 45 50
−Ps(τ)
SCATTERED PULSE
−7 −2 3 8 13 18 23 28 33 38 43 48
τ

Fig. 3.3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Pi(τ)
INCIDENT PULSE
-5 0 5 10 15 20 25 30 35 40 45 50
-Ps(τ)
SCATTERED PULSE
-7 -2 3 8 13 18 23 28 33 38 43 48
τ

DEUXIEME COLLOQUE SUR LA DIFFUSION DES ONDES ULTRASONORES

Université Paris VII

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INVERSE SCATTERING AND ACOUSTIC SPECTROSCOPY

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SUMMARY : Echoes of acoustic or elastic waves scattered from a target carry within them the resonance features caused by the excitation of the eigenvibrations of the target. By means of a suitable background subtraction it is possible to isolate the target's spectrum of resonances. This spectrum characterizes the target just as an optical spectrum characterizes the chemical element or compound that emits it. Extracting the resonance information from the echo allows the possibility of identifying the target as to its size, shape, and composition. This is illustrated by studying the dependence of the resonance spectra of fluid targets upon changes of target shape, including variations from spheres to prolate spheroids and finite-length cylinders. The resulting "acoustic spectroscopy" (a concept introduced by Derem) generates the same type of level scheme as in optics, and it may thus be used for solving some aspects of the "inverse scattering problem" (i.e., the problem of identification of the nature of the target from the returned echoes). A study along these lines shows that the eigenfrequencies of fluid-filled cavities in a solid medium (obtained by us in the complex frequency plane) are tied to resonance features in the scattering amplitude which can be analyzed to provide the material parameters (density and sound speed) of the fluid filler : the resonance spacing giving the sound speed, and the resonance widths the fluid density-hence leading to a solution of the inverse scattering problem in this case.

RESUME : Les échos d'ondes acoustiques ou élastiques diffusés par une cible portent en eux des effets de résonances causés par l'excitation des vibrations propres de la cible. Par le moyen d'une soustraction appropriée du fond, il est possible d'isoler le spectre des résonances de la cible. Un tel spectre caractérise la cible autant qu'un spectre optique caractérise

l'élément ou le composé chimique émetteur de ce spectre. L'extraction des échos de l'information sur les résonances offre la possibilité d'identifier la cible, par sa grandeur, sa forme, et sa composition. Nous illustrons cela en étudiant la dépendance des spectres de résonances de cibles fluides en changeant la forme de la cible, ceci inclus les variations entre une sphère et un sphéroïdeoblong, ou un cylindre de longueur finie. Le concept à suivre de "spectroscopie acoustique" (concept introduit par Derem) a comme base la même sorte de schéma de niveaux que l'on trouve en optique, et il peut alors être utilisé pour résoudre en quelque sorte le "problème inverse" (c'est-à-dire l'identification de la nature de la cible en étudiant les échos). Une telle étude montre que les fréquences propres des cavités dans un milieu solide remplies d'un fluide (obtenues ici dans le plan complexe) sont liées aux effets de résonances dans l'amplitude du signal diffusé, qui peuvent être analysées afin de fournir les paramètres physiques du fluide : l'espacement des résonances donnant la vitesse du son, la largeur des résonances donnant la densité du fluide, menant ainsi à la solution du problème inverse dans ce cas.

1. INTRODUCTION.

In 1979, A. Derem wrote [1] : " L'analyse de la diffusion des ondes acoustiques par un cylindre élastique immergé, fait surgir une véritable spectroscopie acoustique". This was said following an analysis of effects of the resonances of elastic objects, after their excitation by incident sound, as they appear in the scattered echoes, and it seems to be the first formulation of this concept in conjunction with elastic resonance frequency schemes. (The previous term "ultrasonic spectroscopy", and the related early experiments of Gericke [2,3], concern the more general notion of the ultrasonic spectrum reflected from scatterers in solids, and the information on these scatterers contained in the spectrum, not referring specifically to any resonances). The new concept was brilliantly justified experimentally shortly afterwards by Maze, Ripoche and others [4,5] in their pioneering experiments which exhibited in a direct fashion the mechanical eigenvibrations of a submerged elastic cylinder, together with the "level scheme" of the corresponding resonances excited by incident sound, straightforwardly extracted from the reflected echo [6]. Figure 1.1 shows the resonances in the reflected sound of an aluminum cylinder in water, as contained in the total echo (top), and as extracted experimentally (bottom) [7]. The bottom curve represents the "acoustic spectrum" of the cylinder, which appears strikingly similar

to the optical spectrum, or level scheme, of atoms or chemical compounds.

These facts then raise the question whether, as in the optical case, such a measured spectrum can provide information on the source of the spectrum (here, the scattering object). This led to a study of the sensitivity of the acoustic level scheme for internal [8,9] or external resonances [10,11] to changes in the shape of the target. As to the information which can be extracted from the spectrum regarding the consistency of the target, we have carried out illustrative calculations on fluid-filled spherical [12,13] or cylindrical [14] cavities in a solid, or on fluid cylinders in a fluid [15], which showed that in general, for a cavity of known shape, the sound velocity in the filler fluid can be obtained from the frequency values of the resonances (together with the size of the cavity), and the density of the filler fluid from the width of the resonances. Finally, the determination of the physical properties of ocean bottom layers has also been shown to be obtainable [16-18] from the effects of layer resonances contained in the acoustic echo.

The proposed solution of inverse scattering problems by utilizing target resonances represents, of course, only one particular approach towards the solution of inverse problems [19]. Other notable techniques in ultrasonics are the mentioned use of ultrasonic spectroscopy [20], or a method of analysis to determine the geometry of material defects based on the Kirchhoff approximation [21,22].

2. ACOUSTIC RESONANCE LEVEL SCHEME.

A calculation of the eigenfrequencies for prolate fluid spheroids and finite-length fluid cylinders in vacuum, which will approximate the internal resonance frequencies in another, much less dense fluid (except for the imaginary parts of the frequencies, which vanish for ambient vacuum) was carried out [8,9] in order to gauge the sensitivity of the level scheme under charges in shape. For the spheroids, the calculation was carried out by subjecting spheroidal wave function to the appropriate (Dirichlet) boundary condition on the inner surface ; for finite-length cylinders, exact solutions are available. Figure 2.1 shows the eigenfrequency "level scheme" for spheres and infinite cylinders as limiting cases, as well as finite-length cylinders and spheroids (axes ratios b/a = 1.11...and 4.0). The striking "splitting" (lifting of degeneracy) of the limiting-case levels fo the smaller symmetry of finite non-spherical bodies, the azimuthal quantum number no layer being degenerate as for the sphere case, was explained

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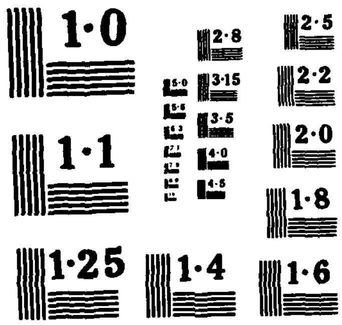

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later [11,23] in terms of the formation of helical surface waves with discrete pitch angles.

For the case of penetrable objects in a fluid or solid medium (including soft or rigid objects where only external resonances appear), one may extend the level scheme of Fig. 2.1. to a plot of resonance frequencies in the complex frequency plane ; see, e.g., Figs. 4.1 or 5.1 of Ref. [23].

3. EXAMPLES OF INVERSE SCATTERING.

It has been shown analytically [12,13] for the example of a fluid-filled spherical cavity in a solid that a determination of the resonance frequencies can determine the sound velocity in the fluid, and of the resonance width the density of the fluid. Numerical calculations for a fluid-filler cylinder in steel [14] bear this out. Figure 3.1 shows the dependence of the reduced resonance frequency spacing $\Delta x_{n\ell}$ (x = ka, k = wave number in the fluid, a = cavity radius) on the ratio c_1/c_2 of internal sound speed c_1 to external p-wave speed c_2, for different values of internal-to-external density ratio, n being the mode number and ℓ the order of the resonance frequency within the n th mode. The near-linear dependence on c_1/c_2, and the intensitivity to ρ_1/ρ_2 indicates the feasibility of determining c_1/c_2 from the resonance spacing.

In Fig. 3.2, we plot the resonance width $\Gamma_{0\ell}$ of the n = 0 resonances vs. $\rho_1 c_1^2/\rho_2 c_2^2$, for different values of c_1/c_2. This illustrates the feasibility of determining ρ_1/ρ_2 from the resonance widths.

4. CONCLUSION

The foregoing represents an elaboration on the concept of Acoustic Spectroscopy, first pronounced by Derem [1], as a means of determining the properties of acoustic targets from the information on the target resonances which is contained in the observed echo returns. The feasibility of this concept, although yet to be generally demonstrated for cases of practical interest [24], appears evident for targets of large impedance contrast with the environment (so that pronounced resonances result). It is also obvious, though, that an application of this approach has to proceed, at this time, with complete understanding of the physics of the scattering process. If this caution is disregarded, failure may easily occur. After sufficient development of the approach, one may ultimately expect, however, that automated systems for experimental acoustic resonance spectroscopy will have been devised.

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FIGURE CAPTIONS.

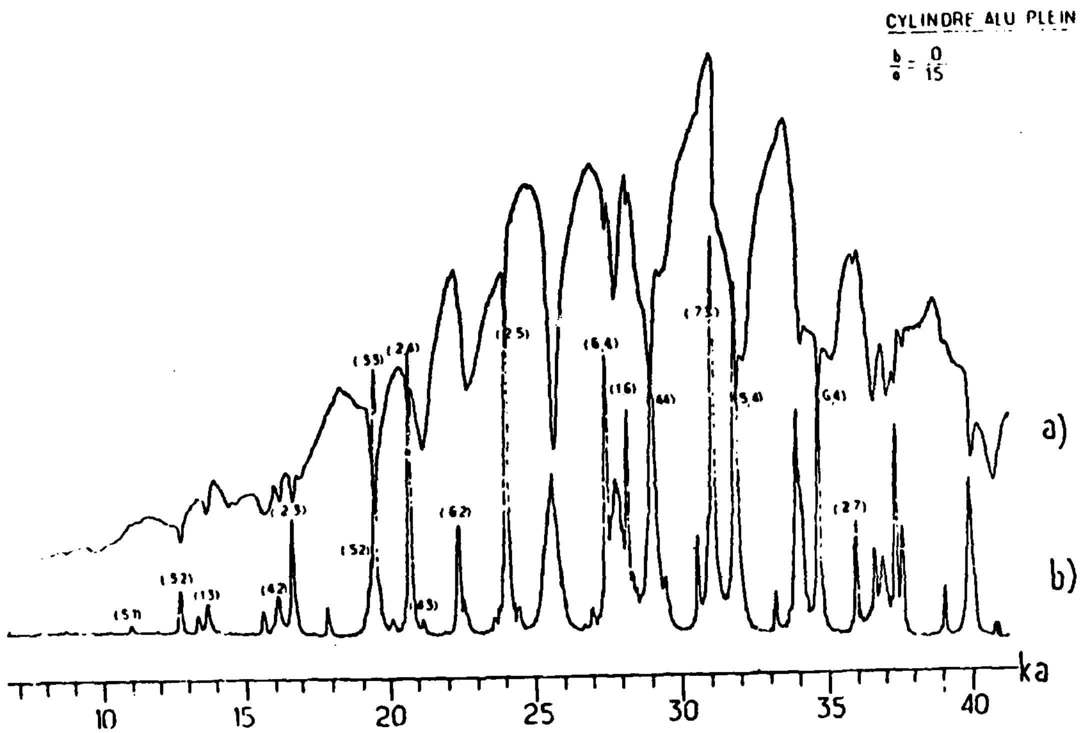
Fig. 1.1. Acoustic backscattering amplitude (total : top ; resonant : bottom) vs. frequency for a solid aluminum cylinder in water.

Fig. 2.1. Eigenfrequency "level scheme" of fluid spheres, cylinders and spheroids in vacuum.

Fig. 3.1. Spacing of the eigenfrequencies of a cylindrical fluid-filled cavity in steel, vs. p-wave speed ratio, for various density ratios.

Fig. 3.2. Resonance widths of a cylindrical fluid-filled cavity in steel, vs. ρc^2 ratio, for various p-wave speed ratios.

Fig. 1.1



CYLINDRE ALU PLEIN
b/a = 0/15
(5 1)
(5 2)
(1 3)
(4 2)
(2 3)
(5 2)
(5 5)
(2 4)
(4 3)
(6 2)
(2 5)
(6 4)
(1 6)
(4 4)
(7 3)
(5 4)
(6 4)
(2 7)
a)
b)
ka
10
15
20
25
30
35
40

Fig. 2.1

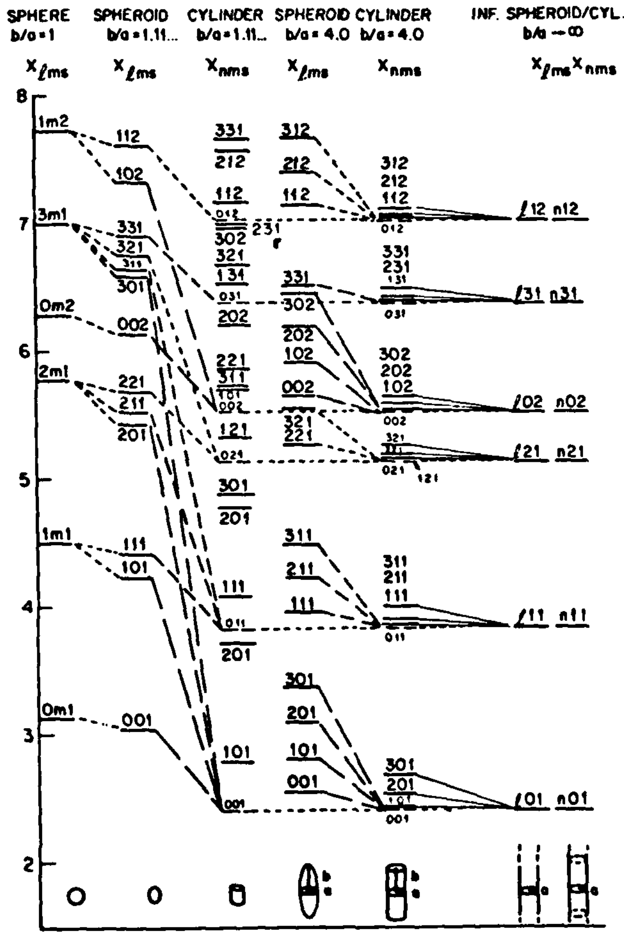
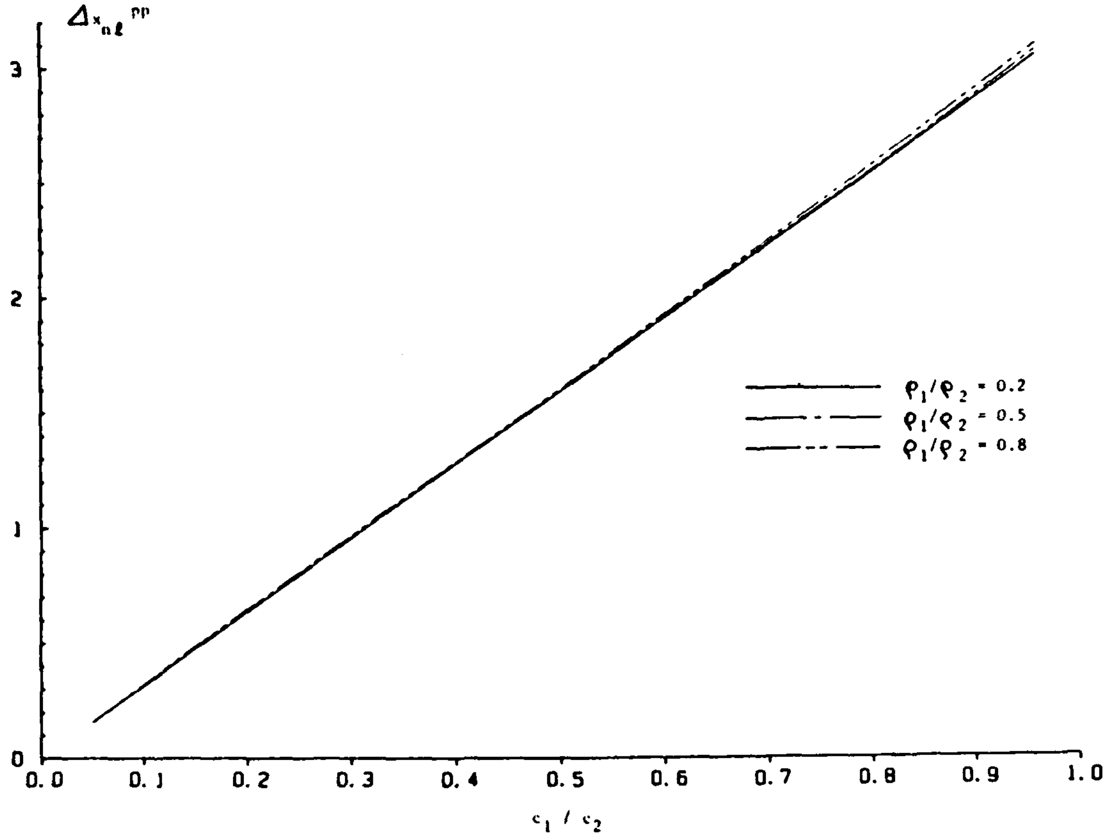
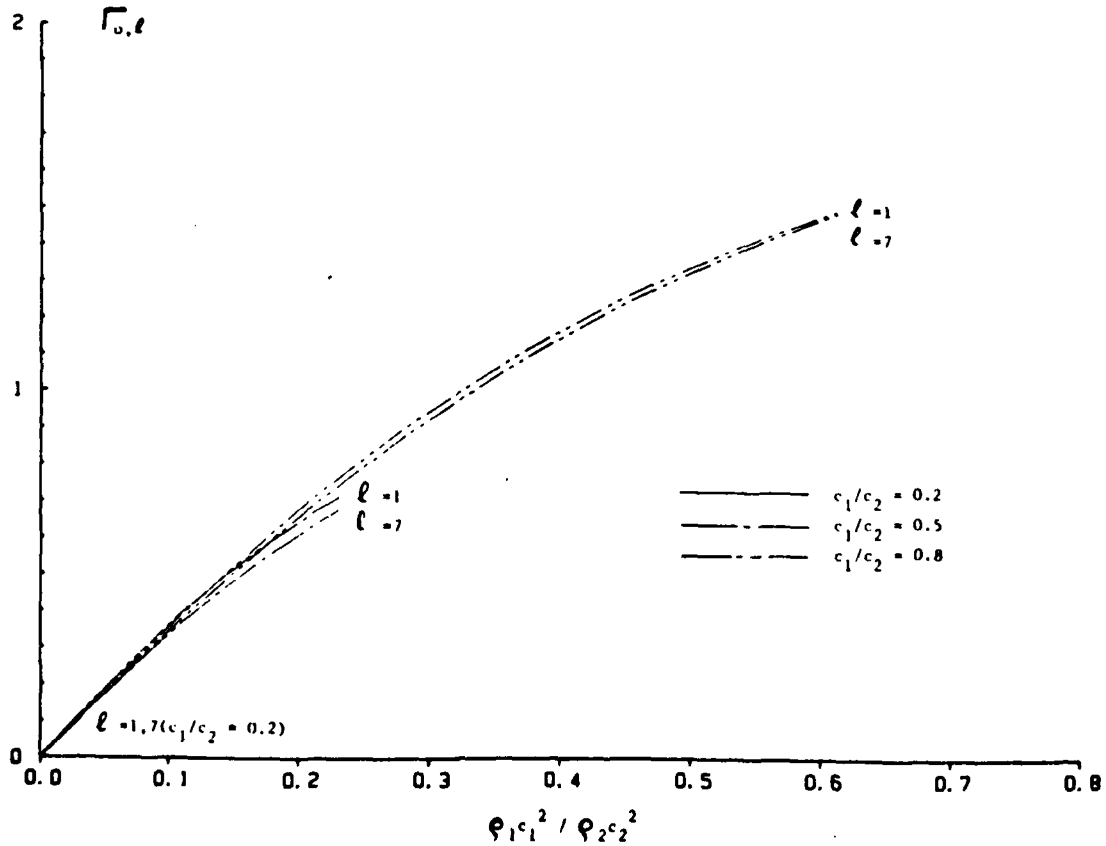

SPHERE b/a=1
SPHEROID b/a=1.11...
CYLINDER b/a=1.11...
SPHEROID b/a=4.0
CYLINDER b/a=4.0
INF. SPHEROID/CYL. b/a→∞
$x_{\ell ms}$
x_{nms}

Fig. 3.1



$\Delta x_{n\ell}^{pp}$
3
2
1
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
c_1 / c_2
$\rho_1/\rho_2 = 0.2$
$\rho_1/\rho_2 = 0.5$
$\rho_1/\rho_2 = 0.8$

Fig. 3.2



$\Gamma_{0,\ell}$
2
1
0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\rho_1 c_1^2 / \rho_2 c_2^2$
$\ell = 1$
$\ell = 7$
$\ell = 1$
$\ell = 7$
$\ell = 1, 7 (c_1/c_2 = 0.2)$
$c_1/c_2 = 0.2$
$c_1/c_2 = 0.5$
$c_1/c_2 = 0.8$

END

FILMED

8-85

DTIC